Contexts, Systems and Modalities: a new ontology for quantum mechanics.

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A. Auffèves & P. Grangier, Found. Phys. 46, 121 (2016) http://arxiv.org/abs/1409.2120
A. Auffèves & P. Grangier, Sci. Rep. 7, 43365 (2017) http://arxiv.org/abs/1610.06164
A. Auffèves & P. Grangier, Phil. Trans. R. Soc. A (2018) http://arxiv.org/abs/1801.01398
P. Grangier & A. Auffèves, Phil. Trans. R. Soc. A (2018) http://arxiv.org/abs/1804.04807
A. Auffèves & P. Grangier, Found. Phys. 50,1781 (2020) http://arxiv.org/abs/1910.13738
N. Farouki & P. Grangier, Found. Sci. 26, 97 (2021) http://arxiv.org/abs/1907.11267
P. Grangier, Found. Phys. 51, 76 (2021) http://arxiv.org/abs/2003.03121
P. Grangier, Entropy 23 (12),1660 (2021) https://arxiv.org/abs/2012.09736
A. Auffèves & P. Grangier, Entropy 24 (2), 199 (2022) https://arxiv.org/abs/2111.10758

Sketch of the talk

1. Einstein, Bohr, Bell, and the experiments

From the EPR-Bohr debate (1935) to loophole-free Bell tests (2015)

2. Can we define a quantum state in physical terms ?

Let us forget about Hilbert space and operators and...

- define a (contextually) objective quantum state then …
- deduce probabilities from quantization.

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3. Contextual objectivity at work :

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Some other issues... and a philosophical outlook

Quantum Physics

* « Quantum Mechanics » elaborated at the end of the 1920's (1925 - 1927 : Schrödinger, Heisenberg, Dirac, Bohr, Born…)

« Greatest intellectual adventure of the 20th century » ?

* **Theory at the basis of our understanding of physical world :** stability and structure of matter, nature of light, interactions between matter and light, superconductivity, superfluidity...

* **Perfectly coherent formalism, huge success, incredible number of applications :** transistor (electronics and computers), laser (telecommunications and internet, medecine, biology…)

* **But... keeps a « mysterious » character :** non-deterministic theory, non-locality (in a subtle way...), no simple correspondance between « quantum objects » and the usual (macroscopic) world.

The Einstein-Bohr debate

* Einstein, Podolsky, Rosen (EPR) 1935 : quantum mechanics is incomplete ("hidden information")

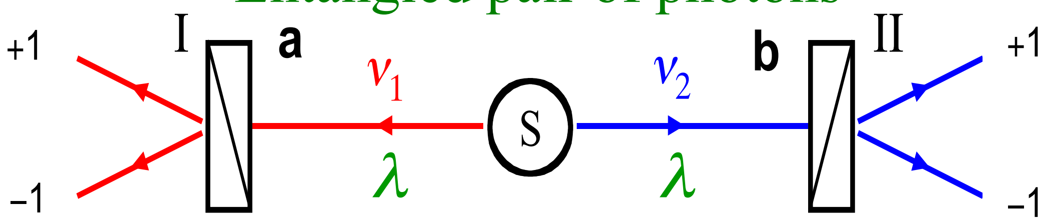
* Bohr disagrees, intense debate over many years but not much attention from majority of physicists

- Quantum mechanics accumulates success:
  - Understanding nature: structure and properties of matter, quantum theory of light, interactions between light and matter...
  - New concepts, and revolutionary inventions: transistor, laser…
  - No disagreement on the validity of quantum predictions, only on its interpretation: debate considered as "philosophical".

The situation changed radically with Bell' theorem (1964) and the acknowledgement of its importance (1969-82...) : One can make experimental tests of « local realism »

Bell's theorem in a nutshell...

Entangled pair of photons



Consider local supplementary parameters theories (in the spirit of Einstein's ideas on EPR correlations):

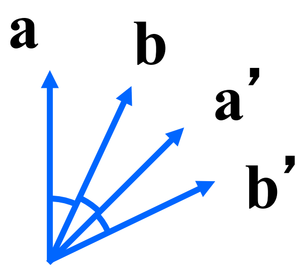
Then the two photons of a same pair have a common property λ

$A(\lambda, \mathbf{a}) = +1 \text{ or } -1 \qquad B(\lambda, \mathbf{b}) = +1 \text{ or } -1 \qquad \rho(\lambda) \geq 0, \ \int \rho(\lambda)\, d\lambda = 1$

Look at the polarization correlation coefficient $E(\mathbf{a}, \mathbf{b}) = (\overline{A\, B})_{\text{av.}\lambda}$ between the measurements results, then (Bell-CHSH inequalities) :

$$-2 \leq S \leq 2 \quad \text{with} \quad S = E(\mathbf{a},\mathbf{b}) - E(\mathbf{a},\mathbf{b}') + E(\mathbf{a}',\mathbf{b}) + E(\mathbf{a}',\mathbf{b}')$$

But... $S_{\text{QM}} = 2\sqrt{2} = 2.828\ldots > 2$



Conflict !
Experiment ?
QM wins !

Experiment with variable polarizers

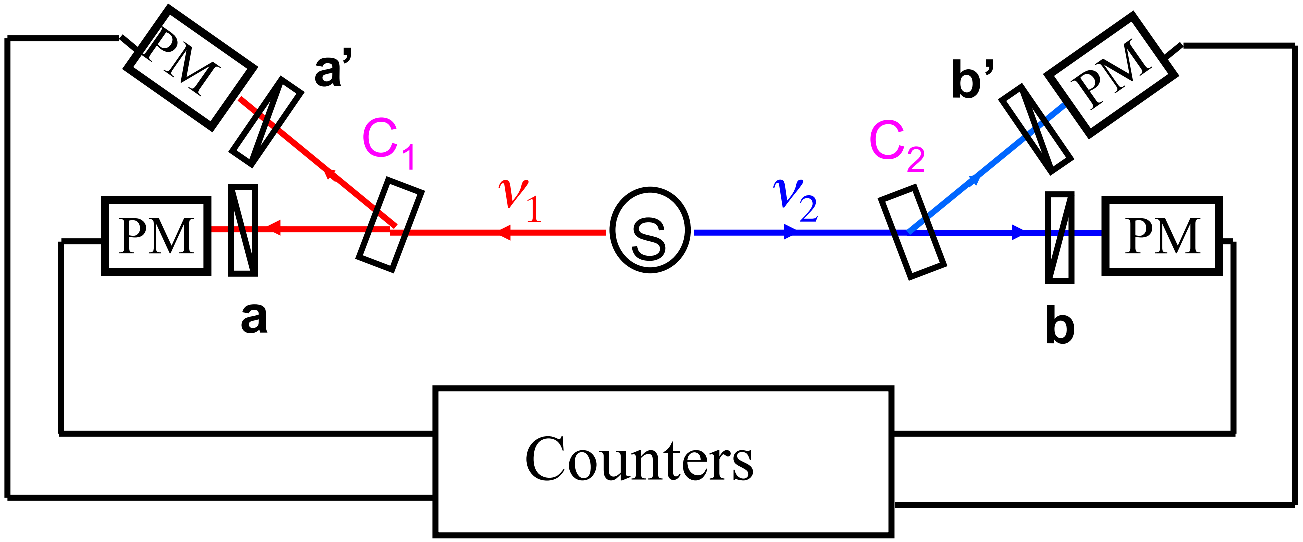
A. Aspect, Phys. Rev. D 14, 1944 (1976).

Bell's locality hypothesis : $\boxed{A(\lambda, \mathbf{a}, \cancel{\mathbf{b}}) \quad B(\lambda, \cancel{\mathbf{a}}, \mathbf{b}) \quad \rho(\lambda, \cancel{\mathbf{a}}, \cancel{\mathbf{b}})}$

In an experiment with variable polarizers (switch faster than L / c), results from relativistic causality (no faster than light influence) !

☹ Not possible with massive polarizer

☺ Possible with optical switches

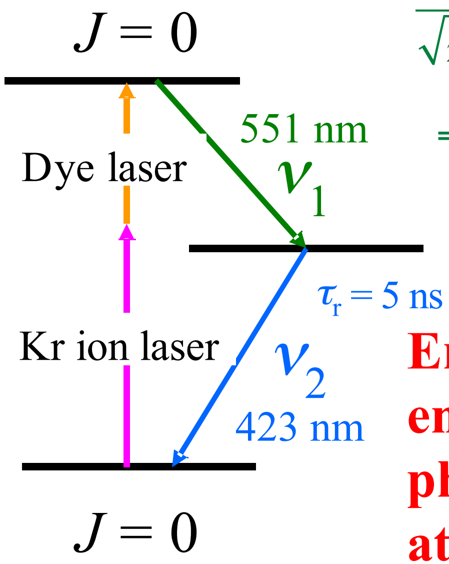


Switches C_1 and C_2 redirects light

- either towards polarizer **a** or **a'**
- either towards polarizer **b** or **b'**

Equivalent to polarizers switching on both sides !

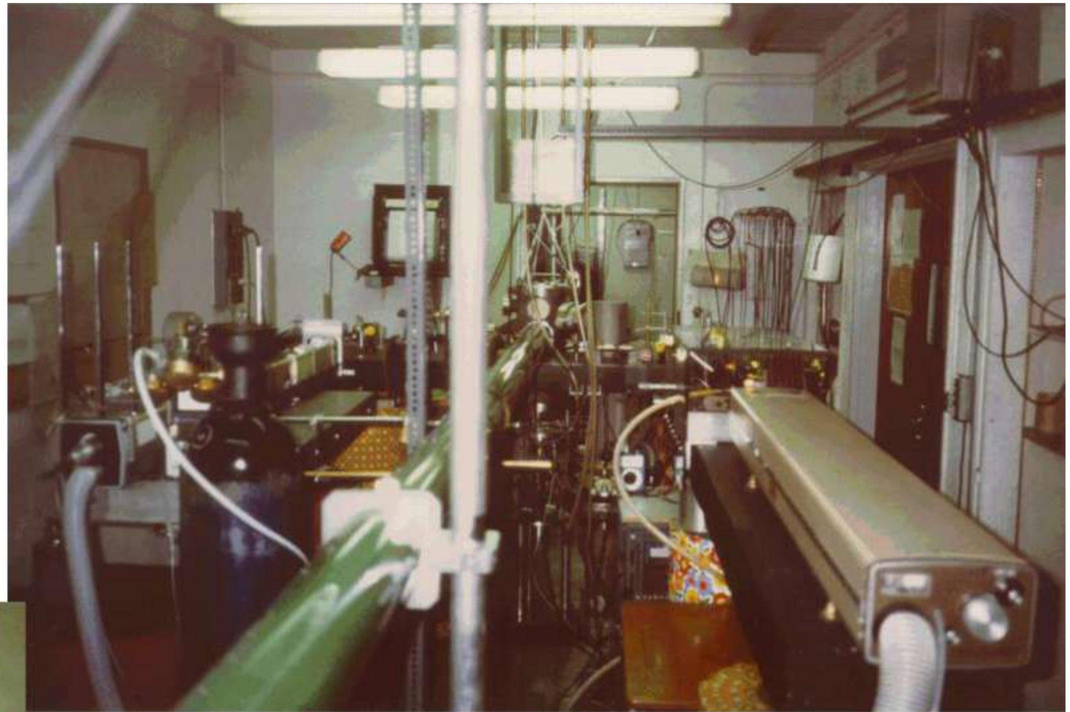
Orsay's source of pairs of entangled photons (1980-82)

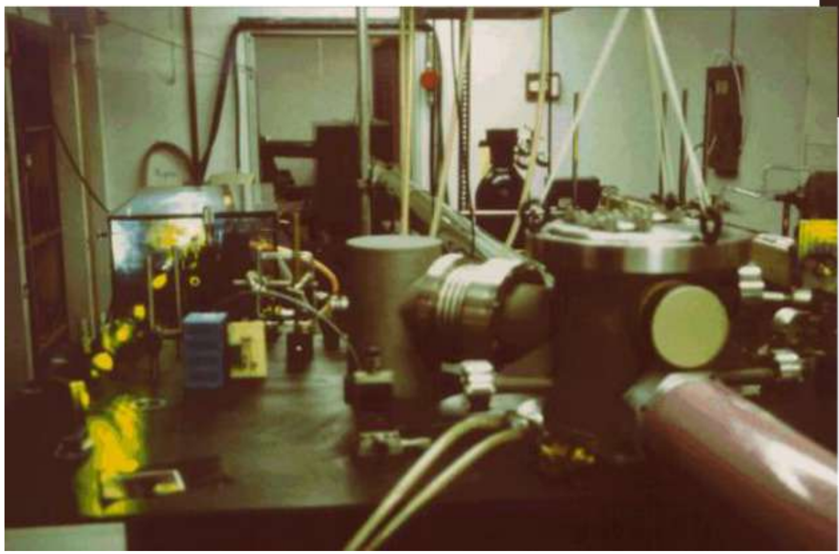


$$\frac{1}{\sqrt{2}}\left\{\left|\sigma_+,\sigma_-\right\rangle+\left|\sigma_-,\sigma_+\right\rangle\right\}$$

$$=\frac{1}{\sqrt{2}}\left\{\left|x,x\right\rangle+\left|y,y\right\rangle\right\}$$

Emission of two entangled photons by an atomic cascade.





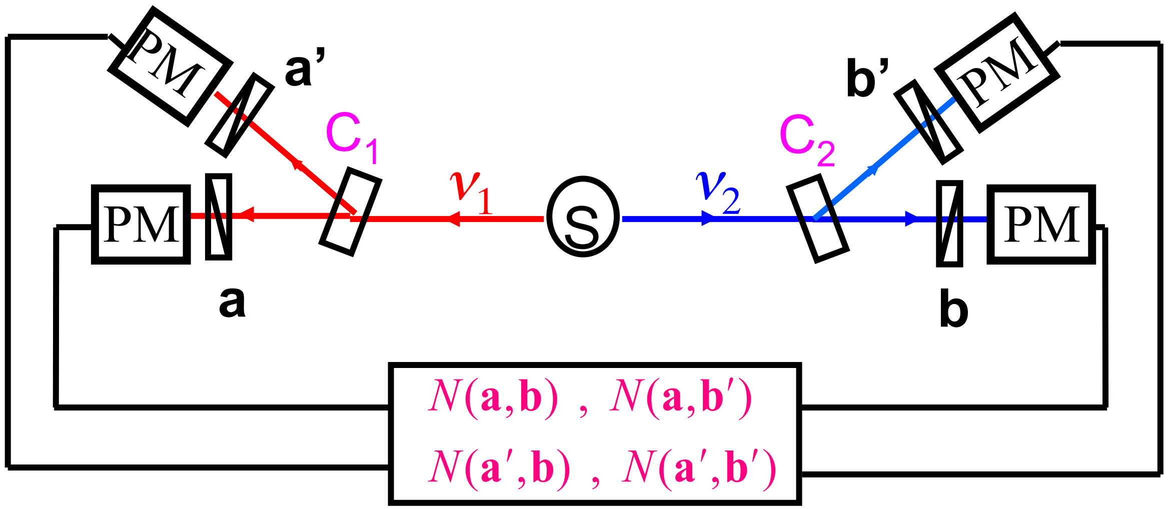
* Laser-induced two-photon excitation of a cascade in a Calcium 40 atomic beam.

☺ 100 detected pairs per second

1% precision for 100 s counting

Experimental results in Orsay

A. Aspect, P. Grangier, G. Roger, PRL 47, 460 (1981)
A. Aspect, P. Grangier, G. Roger, PRL 49, 91 (1982)
A. Aspect, J. Dalibard, G. Roger, PRL 49, 1804 (1982)



"Static polarizers" :
* violation by 40 st. dev. within a few minutes

"Moving polarizers" :
* reduced signal
* several hours counting
* switching uncorrelated but not fully random

... but convincing results :

* Bell's inequalities violated by 6 standard deviations.
* **Each measurement space-like separated from setting of distant polarizer -> Einstein's causality enforced**
* Remaining "loopholes" : low efficiency, imperfect switching

Physics ABOUT BROWSE PRESS

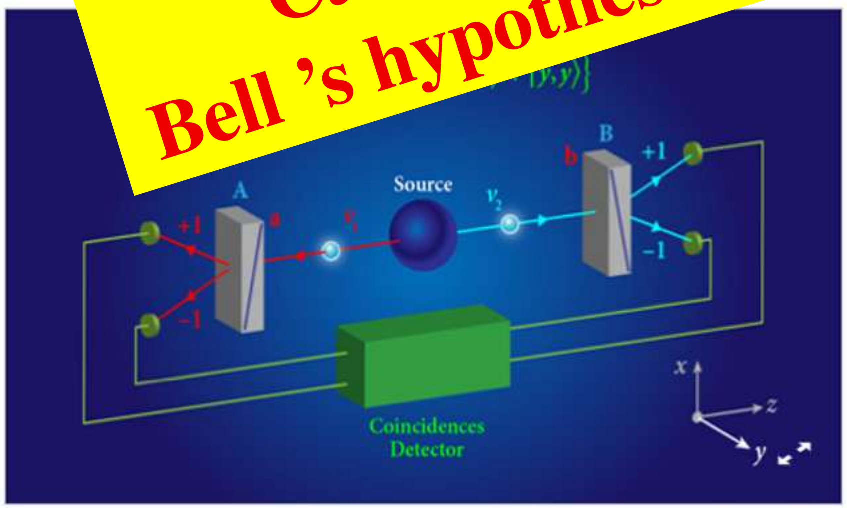
Viewpoint: Closing the Door on Einstein and Bohr's Quantum Debate

Alain Aspect, Laboratoire Charles Fabry, Institut d'Optique Graduate School, …
Palaiseau, France

December 16, 2015 • *Physics* 8, 123

By closing two loopholes at … remove the last doubts that we should renoun… quantum information technologies.

Careful but unavoidable conclusion :
Bell 's hypotheses (local realism) are untenable !



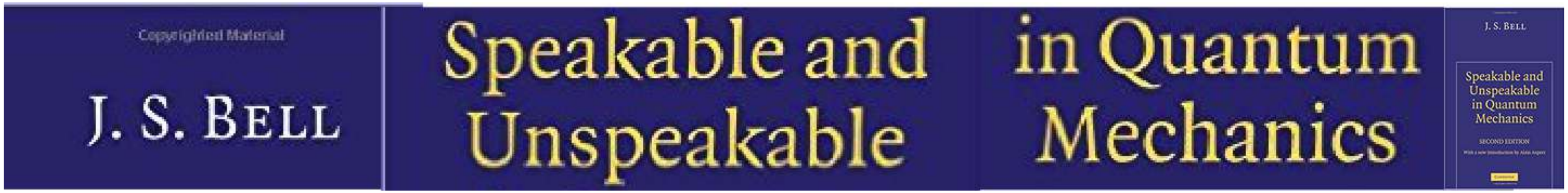
APS/Alan Stonebraker

B. Hensen et al., Nature 526, 682 (2015).
"Loophole-free Bell Inequality Violation Using Electron Spins Separated by 1.3 Kilometres"

M. Giustina et al., Phys. Rev. Lett. 115, 250401 (2015).
"Significant-Loophole-Free Test of Bell's Theorem with Entangled Photons"

L. K. Shalm et al., Phys. Rev. Lett. 115, 250402 (2015).
"Strong Loophole-Free Test of Local Realism"

W. Rosenfeld et al, Phys. Rev. Lett. 119, 010402 (2017).
"Event-ready Bell test using entangled atoms simultaneously closing detection and locality loopholes"



Let us anticipate that quantum mechanics works also for Aspect. How do we stand? I will list four of the attitudes that could be adopted.

(1) ~~The inefficiencies of the counter, and so on, are essential. Quantum mechanics will fail in sufficiently critical experiments.~~

(2) There *are* influences going faster than light, even if we cannot control them for practical telegraphy. Einstein local causality fails, and we must live with this. [must be instaneous: N. Gisin et al, Nat. Phys. 8, 868 (2012)]

(3) The quantities *a* and *b* are not independently variable as we supposed. (...). . Then Einstein local causality can survive. But apparently separate parts of the world become deeply entangled, and our apparent free will is entangled with them.

(4) The whole analysis can be ignored. The lesson of quantum mechanics is not to look behind the predictions of the formalism. As for the correlations, well, that's quantum mechanics.

Philosophical standpoint

Many physicists (including me) will support **Physical Realism**, understood as :
The purpose of physics is to study entities of the natural world, existing independently from any particular observer's perception, and obeying universal and intelligible rules.

Many physicists (inc. me) look at **certain and reproducible events as real,** so we like the definition given by Einstein, Podolsky and Rosen :
If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

but Bell tests show that this view does not work as such... so don't forget Bohr's answer :
The very conditions which define the possible types of predictions regarding the future behavior of the system constitute an inherent element of the description of any phenomenon to which the term "physical reality" can be properly attached.

Could all these statements be made compatible together ?
We will propose an answer later, but remember:

Sir Arthur Conan Doyle (1920's) : Once you eliminate the impossible, whatever remains, no matter how improbable, must be the truth.

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Some other issues... and a philosophical outlook

Classical Physics : Systems and Properties

Classical physics :

* A **« System »** is an entity of the natural world that can be isolated well enough to carry physical properties with definite values, such as mass, charge, position...

* Such properties are measured by using devices external to the system, and attributed to the system itself: **a particle « has » a mass, a position, a velocity...**

* Strictly speaking one should say: ..**when it is measured by this given apparatus.** The complete specification of this apparatus will be called a **« Context »** , but it can be forgotten in classical physics, only the results of the measurement matter.

* Once measured, the properties are « known », they can be measured repeatedly, and the results can be predicted with certainty, taking into account the dynamical evolution of the system : **the properties « belong » to the system (ID card).**

Quantum Physics :
Systems, Contexts, and Modalities

Quantum physics :

* A « **System** » is an entity of the natural world that can be isolated well enough to carry properties with definite values, such as mass, charge, position...

* Such properties are measured by using devices external to the system, and the complete specification of the measurement apparatus will be called a **« Context »**

*** Once measured, the values of the properties can be measured repeatedly, and the results can be predicted with certainty, in a given context.**

*** The set of definite (fully predictable) values of the physical properties belongs jointly to the system and the context, and it will be called a modality.**

***What is « real » is the combination of Context, System and Modality (CSM)**

Usual langage (classical...) : a photon « has » a polarization oriented at 45°
CSM : the photon (system) is transmitted with certainty (modality) through a polarizer oriented at 45° (context)

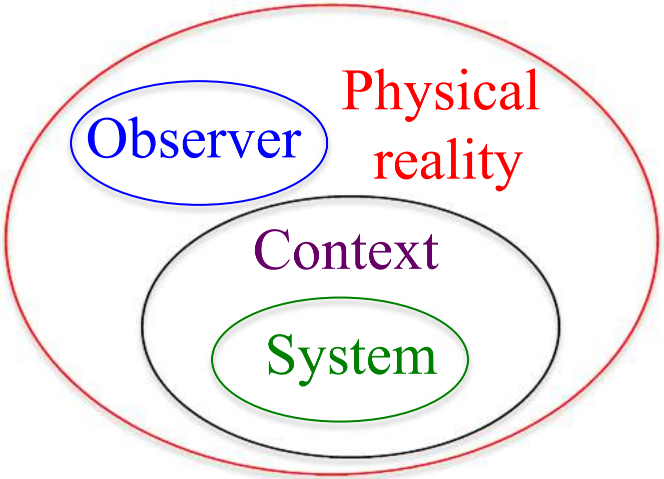
Element of physical reality vs modality

If, without in any way disturbing a system **neither changing the context,** we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element physical reality corresponding to this physical quantity. **It is called a modality.**

* This statement agrees with both the **« certainty » required by Einstein and the « very conditions » required by Bohr** to make and to check definite and reproducible predictions (i.e. with objectivity, taken as contextual).

* Therefore **the « object »** carrying the element of physical reality **is a system within a context.**

* The « split » between system and context is not a problem for CSM, because a modality is defined in terms of both the system and the context, and the system **cannot include the context.**



« Quantum mechanics can explain anything, but not everything »

A. Peres and W. H. Zurek, Am. J. Phys. 50, 807 (1982)

The Quantization Principle

What prevents to have a unique context where all modalities would be defined ? (this would be back to classical physics)

1. Within a given context, the modalities are mutually exclusive, i.e. if one of them is realized (or true), the other ones are not realized (or wrong). In a different context, there will be a different set of mutually exclusive modalities.

2. Modalities taken from two different contexts are generally « non mutually exclusive », or « incompatible », i.e. if one one of them is realized, one cannot tell whether the other ones are realized or not. Incompatible modalities are « non-classical » : classically, it should be possible to distinguish them by making more measurements, i.e. by extending the context.

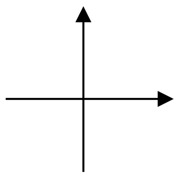
3. Quantization principle : the number N of mutually exclusive modalities is a property of the quantum system, and it is independant of the context.

Example of polarized photons

2 exclusive modalities

context

« + »

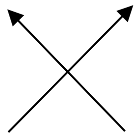


cat

dog

context

« x »



black

white

4 classical cases :

black cat

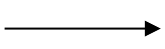
black dog

white cat

white dog

1. context

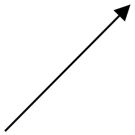
« + »



Select cat

2. context

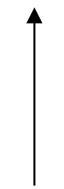
« x »



Select black cat

3. context

« + »



Select dog ? yes !

Classical conclusion: Half of black cats are actually dogs !

Why nonsense ?

Because « black » and « cat » are **non-exclusive modalities** (cannot be combined)

Usual langage (classical...) : the photon « has » a polarization oriented at 45°

CSM : the photon (system) is transmitted with certainty (modality) through a polarizer oriented at 45° (context)

Why probabilities ?

The quantization principle requires that one must use probabilities !

Given one system and two contexts C and C', each with N modalities, **combining the incompatible modalities** b_n from C and b_m' from C' in a single context with more than N modalities **is forbidden by the quantization principle.**

Ex. for N=2	Vertical polarizer Tv or Rv Two mutually exclusive modalities, ok	Diagonal polarizer Td or Rd Two mutually exclusive modalities, ok	Combined results TvTd, TvRd, RvTd, RvRd Four modalities, not a context if N = 2.

Therefore the only relevant question that can be answered by the theory is:
if the initial modality is b_n in context C, what is the probability for obtaining modality b_m' from a quantum measurement in context C' ?

Probabilities are not related to any ignorance, but to the ontology of the theory :
results taken from different contexts cannot be put together to get more details, because one would get more than N mutually exclusive sub-modalities.

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Some other issues... and a philosophical outlook

The CSM physical axioms

Axiom 1 (modalities)

(i) Given a physical system, a **modality** is defined as the values of a complete set of physical quantities that can be predicted with certainty and measured repeatedly on this system.

(ii) Here "complete" means the largest possible set compatible with certainty and repeatability, for all possible modalities attached to this set. **This complete set of physical quantities is called a context, and a modality is attributed to a system within a context.**

(iii) Modalities in different contexts may be connected with and certainty (**extracontextuality**)

Axiom 2 (contextual quantization)

(i) For a given context, there exist N distinguishable modalities, that are mutually exclusive: if one modality is true, or realized, the others are wrong, or not realized.

(ii) The value of N, called the dimension, is a characteristic property of a given quantum system, and is the same in all relevant contexts.

Axiom 3 (changing contexts)

Given axioms 1 and 2, the different contexts relative to a given quantum system are related between themselves by continuous transformations which are associative, have a neutral element (no change), and an inverse. Therefore **the set of context transformations has the structure of a continuous group.**

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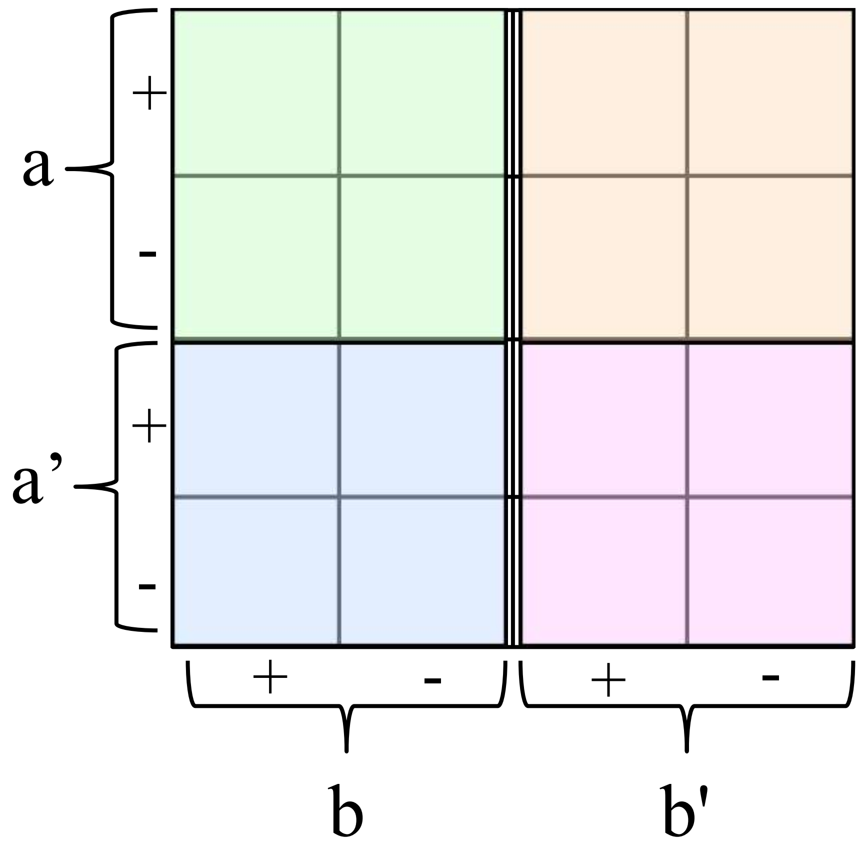
Modalities in a Bell experiment

Centre National de la Recherche Scientifique

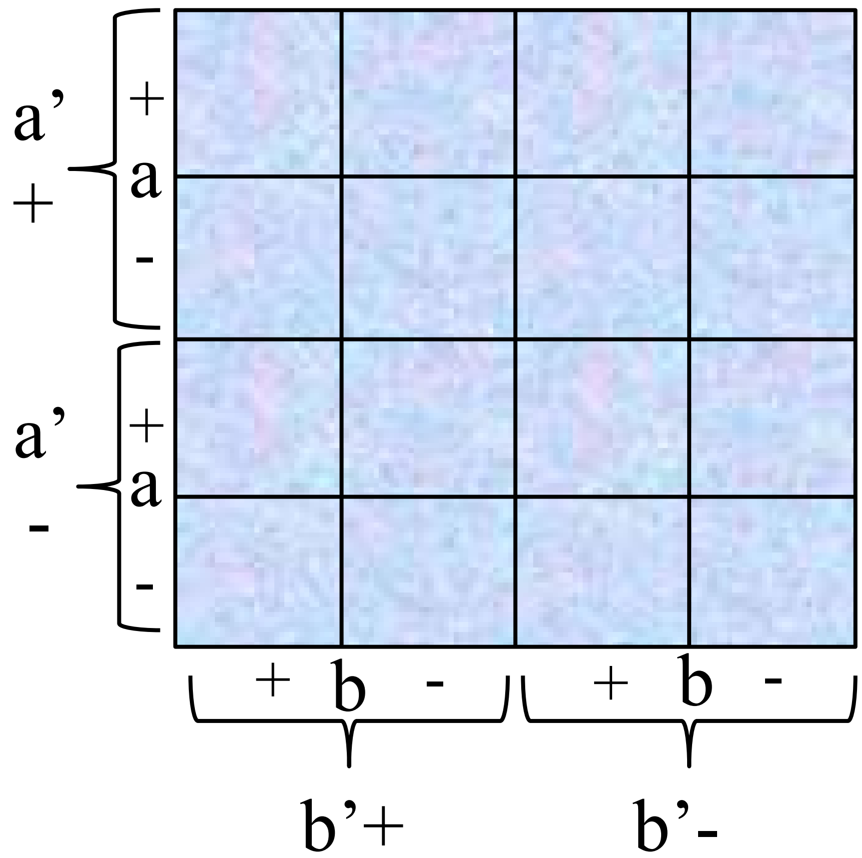
4 different contexts : MQ



N = 4 mutually exclusive modalities in each context

Violation of Bell’s ineq. : ok !

Global context : classical

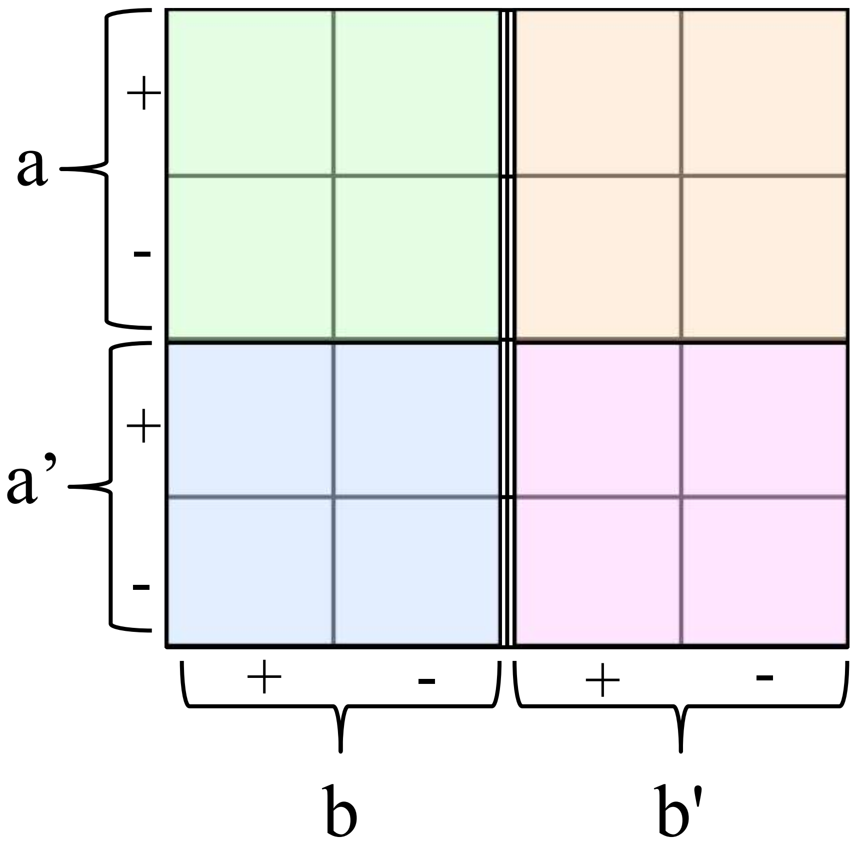


16 mutually exclusive modalities in a global context

Obeys Bell’s ineq. : no !

Modalities in a Bell experiment

4 different contexts : MQ



Crucial observation : The certainty of a modality can be transferred between different contexts : extracontextuality !

4 other different contexts : MQ

$\|+,-\rangle$	$\|-,+\rangle$
$\|+,+\rangle$	$\|-,-\rangle$
$\|1,1\rangle$	$\|1,-1\rangle$
$\|1,0\rangle$	$\|0,0\rangle$
$\|\Psi^+\rangle$	$\|\Psi^-\rangle$
$\|\Phi^+\rangle$	$\|\Phi^-\rangle$

S_{z1}, S_{z2}
$m_1 = \pm 1/2$
$m_2 = \pm 1/2$

Total spin $\mathbf{S^2}$, S_z
$S = 0,1$
$m = -1,0,1$

Bell states
$|\Phi^\pm\rangle = (|+,+\rangle \pm |-,-\rangle)/\sqrt{2}$
$|\Psi^\pm\rangle = (|+,-\rangle \pm |-,+\rangle)/\sqrt{2}$

Extra-contextuality and extravalence

Definition : When a system interacts in succession with different contexts, certainty and repeatability can be transferred between their modalities. This is called **extracontextuality**, and this defines an equivalence class between modalities, called **extravalence** (it is reflexive, symmetric, transitive).

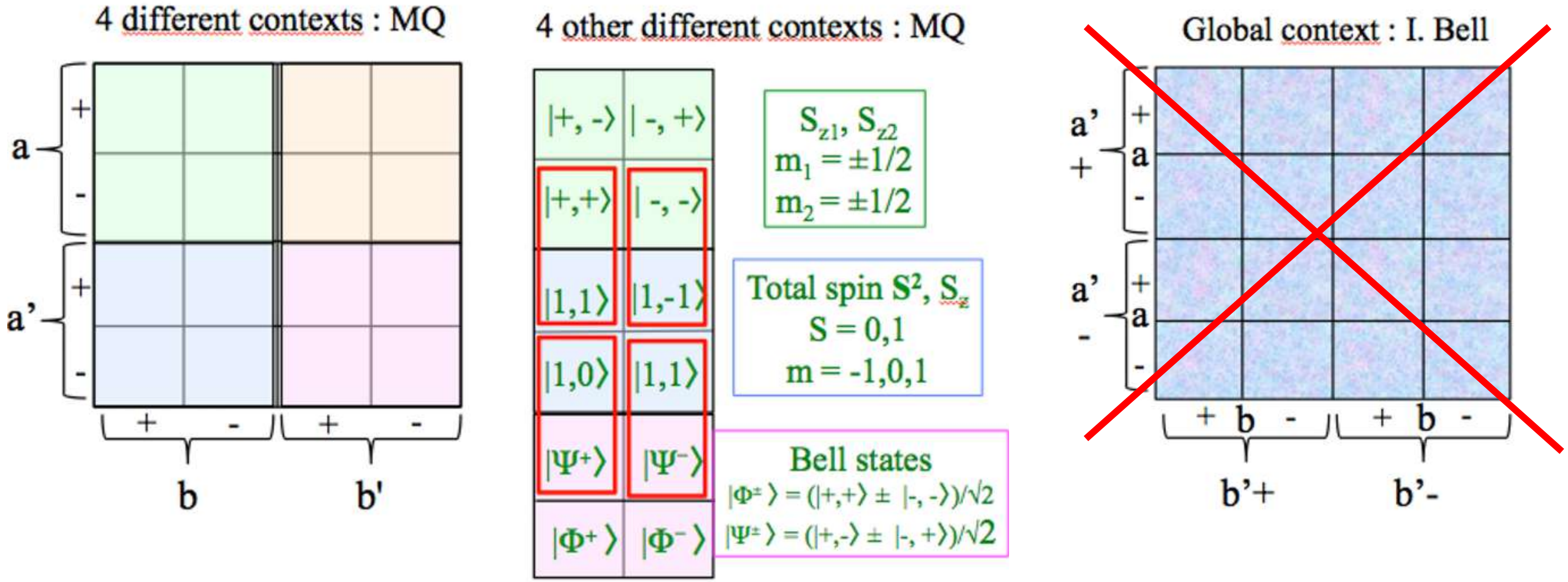
Theorem : Given an initial modality and context, the probability to get another modality in another context keeps the same value as long as the initial and final extravalence classes remain the same.

$$\begin{tikzcd} u_i \arrow[r, dashed] \arrow[dr, dashed] \arrow[d, leftrightarrow] & v_j \arrow[d, leftrightarrow] \\ x_l \arrow[r, dashed] \arrow[ur, dashed] & w_k \end{tikzcd}$$

The modalities u_i , v_j , x_l , w_k belong to four different contexts, and u_i is extravalent with x_l , resp. v_j with w_k (full lines). Then all probabilities represented by dashed lines are equal.

⇒ extravalent modalities embed the idea of non-contextuality of probability assignments: the probability belongs to the extravalence class, not to the modality.

Modalities in a Bell experiment



Heuristics : how can we make sure that

- there are only N mutually exclusive modalities in any context ?
- the certainty of a modality can be transferred between contexts ?

Born's rule : the CSM way (1)

Inductive part : use projectors !

Heuristics : how can we make sure that

- there are only N mutually exclusive modalities in any context ?

- the certainty of a modality can be transferred between contexts ?

Let's attribute a N x N projector to an extravalence class, with

- **orthogonal projectors ⇔ mutually exclusive modalities (in a context)**
- **same projector ⇔ mutually certain modalities (in an extravalence class)**

- the probability to find a given result (reproducible with certainty after being found) given an initial 'state' is a function $f(P_n)$, where f depends only on the initial state, and $P_n = |\psi_n\rangle\langle\psi_n|$ is a projector associated with the result.

- the probabilities are additive for mutually orthogonal (commuting) projectors, and $\sum_n f(P_n) = 1$ for any orthogonal set such that $\sum_n P_n = \mathrm{Id}$

Born's rule : the CSM way (2)

Deductive part : recovering the usual QM formalism

- Theorem (Uhlhorn) : unitary transformations between contexts.

Consider two contexts Cp (with N mutually orthogonal projectors Pi), Cq (with N mutually orthogonal Qj). Expressing the Pi as a function of the Qj when changing the context **must preserve the orthogonality of the Pi** : then it must be a unitary or antiunitary transformation (Uhlhorn's theorem). We want also to connect continuously the context change with the identity (no change of context, Cp = Cq) : **unitary transformation only.**

- Theorem (Gleason) : Born's rule.

The previous requirements fit with the hypotheses of Gleason's theorem :

- if the probability 1 is reached when changing contexts then one gets

  Born's rule for pure states, p(j | i) = Trace(Pi Qj).

- otherwise one gets Trace(ρ Qj) where ρ is a density matrix.

Open Access Article

Revisiting Born's Rule through Uhlhorn's and Gleason's Theorems

by Alexia Auffèves [1] and Philippe Grangier [2,*]

Entropy **2022**, *24*(2), 199; https://doi.org/10.3390/e24020199

Theorem 1 (Uhlhorn's theorem [20,21]). *Let $\mathcal{H}$ be a complex Hilbert space with $dim(\mathcal{H}) \geq 3$, and let $P_1(\mathcal{H})$ denote the set of all rank-one projections on $\mathcal{H}$. Then, every bijective map Γ: $P_1(\mathcal{H}) \to P_1(\mathcal{H})$, such that $pq = 0$ in $P_1(\mathcal{H})$ if and only if $\Gamma(p)\Gamma(q) = 0$, is induced by a unitary or anti-unitary operator on the underlying Hilbert space.*

Theorem 2 (Gleason's Theorem [23,24]). *Let f be a function to the real unit interval from the projection operators on a separable (real or complex) Hilbert space with a dimension at least 3. If one has $\sum_i f(P_i) = 1$ for any set $\{P_i\}$ of mutually orthogonal rank-one projectors summing to the identity, then there exists a positive-semidefinite self-adjoint operator ρ with unit trace (called a density operator), such that $f(P_i) = Trace(\rho P_i)$.*

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Unscrambling the quantum omelette*

… by unscrambling physics from mathematics ?

Modality **Real physical phenomenon, involving a system and a context.**	$\neq$	**State vector or projector** **Mathematical object, associated to a class of extravalent modalities.**

* This is related to Bohr's view, but also major differences : **for CSM the « explanation » of quantum behaviour is quantization** (Rovelli, Zeilinger...)

* Contextual quantization implies that **modalities are related probabilistically between different contexts** (probabilities are the only way to manage quantization)

* Modalities are neither fully contextual nor non-contextual, they are **extra-contextual** (i.e. the certainty of modalities can be transferred between contexts).

* Given that, the QM formalism is a **mathematical way to calculate these probabilities**, consistently with the CSM axioms.

* This wording is taken from E.T. Jaynes, and it has also been used by Christian de Ronde, Karl Svozil, Rob Spekkens... and maybe others.

Thank you for your attention !

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Thank you to Franck Laloë, Roger Balian, Anthony Leverrier...

Philosophical options...

Hard subjectivist ("crazy bayesian")

- how certain are you that you are certain ?
- a probability assignment is not a fact (Caves, Fuchs and Schack...)
- **the fact** : it is possible to design a set of measurements so that if you perform it again and again on the same system it will give again and again the same result.

In such a case we tell that the system is in a well defined modality / quantum state.

Hard realist ("deceived lover of hidden variables")

- a state corresponds to a set of elements of physical reality (= the results can be predicted with certainty and measured without changing in any way the system).
- **the fact :** reality is ok, but it must be attributed **jointly** to the context and the system; then a modality is a quite acceptable element of physical reality, and it gives a meaning to « non locality without any spooky action at a distance ».

Philosophical options...

Hard platonist ("mathematical objects do exist")

- a vector in an Hilbert space is not a mathematical tool, but a definition of reality
- unitarity of evolution is basic, the observed classical world must "emerge" from it
- **the fact** : manipulating vectors (projectors) in an Hilbert space is the quantum way to calculate probabilities, it is not a "reality". **The "reality" is the modality, i.e. the set of values of physical properties that you will obtain again and again by performing measurements on the same system in the same context.**

Super-hard platonist ("mathematical objects exist physically")

- nothing else than $|\psi\rangle$ exist (within a universal $|\psi\rangle$)
- the many-world picture must be understood ontologically: there are many « me »
- **the fact** : same as above, but here it is not recognized as a fact, since the only "facts" are about $|\psi\rangle$ itself, so there is no way to agree (physical realism is gone).