Contextual inferences, nonlocality, and the incompleteness of quantum mechanics.

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Viewpoint: Closing the Door on Einstein and Bohr’s Quantum Debate

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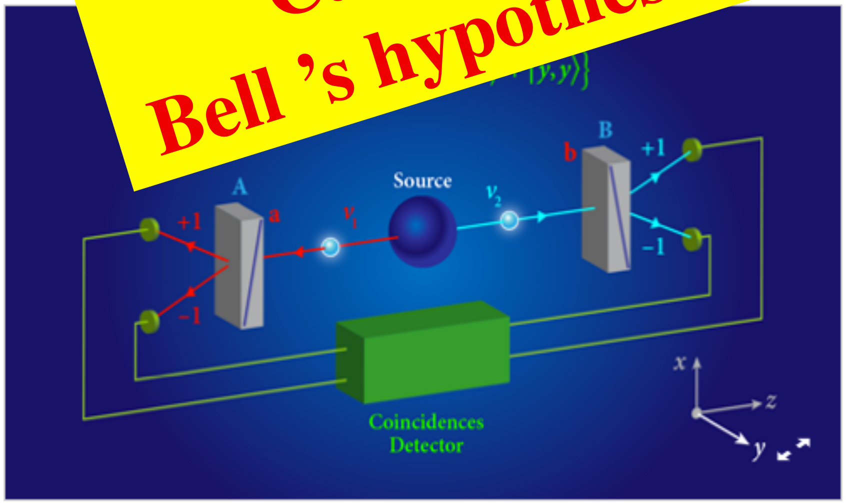
By closing two loopholes at … remove the last doubts that we should renoun… …uantum information technologies.

Careful but unavoidable conclusion :
Bell ’s hypotheses (local realism) are untenable !



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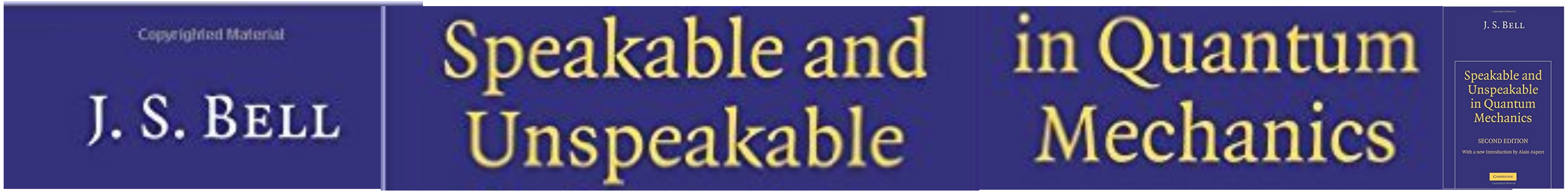


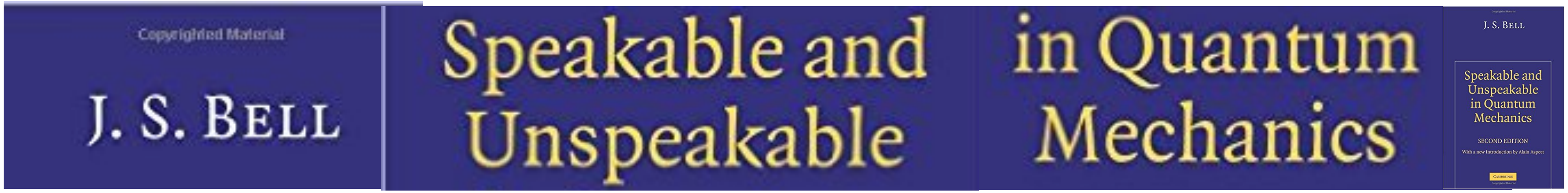
Let us anticipate that quantum mechanics works also for Aspect. How do we stand? I will list four of the attitudes that could be adopted.

(1) ~~The inefficiencies of the counter, and so on, are essential. Quantum mechanics will fail in sufficiently critical experiments.~~

(2) There *are* influences going faster than light, even if we cannot control them for practical telegraphy. Einstein local causality fails, and we must live with this. [must be instaneous: N. Gisin et al, Nat. Phys. 8, 868 (2012)]

(3) The quantities *a* and *b* are not independently variable as we supposed. (...). . Then Einstein local causality can survive. But apparently separate parts of the world become deeply entangled, and our apparent free will is entangled with them.

(4) The whole analysis can be ignored. The lesson of quantum mechanics is not to look behind the predictions of the formalism. As for the correlations, well, that's quantum mechanics.



Go one step back, and give a mathematical content to options 2/3/4 :

(2) There *are* influences going faster than light, even if we cannot control them for practical telegraphy. Einstein local causality fails, and we must live with this.

=> Violation of « elementary locality », without faster than light signalling.

(3) The quantities *a* and *b* are not independently variable as we supposed. and *B*. Then Einstein local causality can survive. But apparently separate parts of the world become deeply entangled, and our apparent free will is entangled with them.

=> Violation of « free choice », there are no random independant events

(4) The whole analysis can be ignored. The lesson of quantum mechanics is not to look behind the predictions of the formalism. As for the correlations, well, that's quantum mechanics.

=> Violation of « predictive completeness » : this cannot be formulated in classical or deterministic framework, but makes sense in a quantum one !

Predictive incompleteness.

Predictive (in)completeness : introduced by Jon Jarrett in "On the physical significance of the locality condition in the Bell arguments", Noûs 18, 569 (1984), not fully exploited at that time.

Usual claim, $|\psi\rangle$ is complete : given a state vector $|\psi\rangle$, one can calculate the probability $|\langle\phi|\psi\rangle|^2$ of getting any other state vector $|\phi\rangle$ in a measurement.

Unusual claim, $|\psi\rangle$ is predictively incomplete : given $|\psi\rangle$, one needs to specify a full measurement in order to obtain a meaningful probability distribution over a set of mutually exclusive events ; so one needs not only $|\phi\rangle$, but also a non-degenerate operator admitting $|\phi\rangle$ as an eigenstate : **this is a context !**

Basic framework (1).

• We use a general framework for conditional probabilities, valid both for usual quantum mechanics (QM) and for so-called hidden variable theories. Look where these two descriptions split, and why.

• EPR-Bohm-Bell scheme : polarizations measurements on entangled photon pairs, described by some λ in a variable space Λ. Alice and Bob carry out measurements with respective polarizers' orientations x and y, and get binary results $a = \pm 1$ and $b = \pm 1$.

• Without loss of generality one can write the following relation between conditional probabilities, by conditioning on λ in some a priori unknown hidden variable space Λ

$$P(ab|xy) = \sum_{\lambda \in \Lambda} P(ab|xy\lambda)P(\lambda|xy) \tag{1}$$

Basic framework (2).

$$P(ab|xy) = \sum_{\lambda \in \Lambda} P(ab|xy\lambda) P(\lambda|xy) \qquad (2)$$

• Here λ specifies whatever may be specified about the emission of the photon pair, in a given shot. This may include variables that fluctuate from shot to shot, and other variables that don't.

• The relation above is true for QM, where the variable space Λ contains only one λ describing the initial state of the entangled pair (e.g. a singlet state). It is also true in demonstrations of Bell's inequalities using stochastic variables (Clauser-Horne). There are differences however, that will come later !

• It is even true for theories where a and b are deterministic functions of λ, x, y ; however the issue of determinism has important consequences, that will show up below.

Introducing physical hypotheses.

A first requirement, sometimes called "freedom of choice", is that the way photons are emitted (λ) does not depend on the variables (x, y) representing Alice's and Bob's measurement settings.

This implies $P(\lambda|xy) = P(\lambda)$, and thus :

$$P(ab|xy) = \sum_{\lambda \in \Lambda} P(ab|xy\lambda)P(\lambda) \qquad (3)$$

Note : this hypothesis is not accepted by "superdeterministic" theories, where λ, x, y are related by some common past. Then there is no independent randomness, and we exclude this option.

For a given initial state λ of the pair a relevant theory should provide $P(ab|xy\lambda)$, so we focus now on this conditional probability.

Basic rules of inference.

Without any further assumptions, one can always write :

$$P(ab|xy\lambda) = P(a|xy\lambda)P^a(b|xy\lambda a) = P^b(a|xy\lambda b)P(b|xy\lambda) \quad (4)$$

where the two decompositions refer respectively to Alice and Bob.

More precisely, on Alice's side ^{a} :

* $P(a|xy\lambda)$: probability for Alice to get result a for input x
* $P^a(b|xy\lambda a)$: probability for Bob to get result b for input y, calculated by Alice who knows x and a

whereas on Bob's side ^{b} :

* $P(b|xy\lambda)$: probability for Bob to get result b for input y
* $P^b(a|xy\lambda b)$: probability for Alice to get result a for input x, calculated by Bob who knows y and b

Elementary locality.

A second meaningful requirement is that the choice of measurement by Alice (resp. Bob) should not have an influence on the result by Bob (resp. Alice). This implies that

$$P(a|xy\lambda) = P(a|x\lambda) \qquad \text{and} \qquad P(b|xy\lambda) = P(b|y\lambda).$$

We will call this condition "elementary locality" (EL), meaning that it is fulfilled for each given λ. As a consequence one has

$$\begin{aligned} P(ab|xy\lambda) &= P(a|x\lambda)P^a(b|y\lambda, xa) \\ &= P^b(a|x\lambda, yb)P(b|y\lambda) \end{aligned} \tag{5}$$

All this agrees with QM, and Bell's inequalities cannot be obtained yet ! Correspondingly, if interpreted "à la Bell", xa and yb in eq.(5) should not be there. Yet, Alice calculates a probability for Bob's result, by using only what is locally available to her ; this does not entail any influence from one side onto the other side.

Contextual inferences vs Bell's hypotheses (1).

Given eq.(5), there is still a missing step to reach Bell's theorem. In order to identify it, let us recall that locality "à la Bell" can be seen as a conjunction of two conditions.

• A first condition is "elementary locality" (EL), already seen :

(EL) $P(a|xy\lambda) = P(a|x\lambda)$ and $P(b|xy\lambda) = P(b|y\lambda)$, which agrees with QM and relativistic causality as explained before.

• A second condition, "predictive completeness" (PC), reads :

(PC) $P^a(b|axy\lambda) = P(b|xy\lambda)$ and $P^b(a|bxy\lambda) = P(a|xy\lambda)$, and it will be interpreted physically below.

Taken together (EL) and (PC) imply the Bell's factorization condition $P(ab|xy\lambda) = P(a|x\lambda)P(b|y\lambda)$, and hence Bell's inequalities.

Since QM agrees with (EL), it must violate (PC).

Note : (EL) is sometimes called parameter independence, and (PC) outcome independence.

Contextual inferences vs Bell's hypotheses (2).

To violate BI while respecting elementary locality, QM must violate predictive completeness, i.e. :

$$P^a(b|xa\,y\lambda) \neq P(b|x\,y\lambda) = P(b|y\lambda)$$

$$P^b(a|yb\,x\lambda) \neq P(a|y\,x\lambda) = P(a|x\lambda).$$

Given this situation, a main step forward is to consider that the description given by λ (or ψ in the quantum case) is **not** complete, and that knowing (xa) does bring something new to Alice : then condition (PC) is violated, by **Alice making a "contextual inference" about Bob's result** (same for Bob about Alice).

In order to make sense of this idea, it is essential to realize that
(i) contextual inference is a non-classical phenomenon, and
(ii) it agrees with relativistic causality, as we explain now.

Contextual inferences are non-classical.

In classical physics, condition (PC) as defined above is verified, and Bell's factorization condition follows. But in quantum physics, knowing Alice's measurement and result allows her to predict more, without invoking any action at a distance.This is because $\lambda \equiv \psi$ does not tell which measurements will be actually carried out by Alice and Bob, **and thus $\lambda \equiv \psi$ is predictively incomplete.**

Adding this information where and when it is **locally** available improves Alice's prediction about Bob's result, and Bob's about Alice's, in agreement with eq.(5), showing the suitability of the concept of contextual inference. This effect does not show up in classical physics, because a classical λ is complete ; but it does show up in QM, because a quantum ψ is (predictively) incomplete, as long as a measurement context has not been specified.

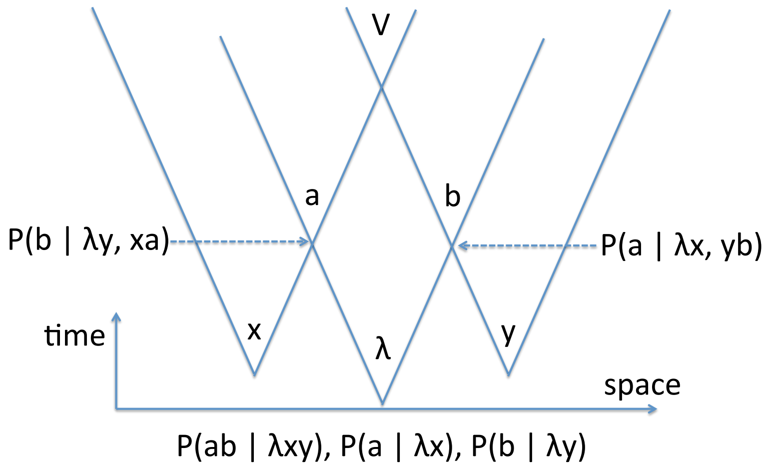
Contextual inferences agree with relativistic causality.

Since contextual inference only applies to the probabilities in eq.(5), it does not involve any physical interaction outside light cones ; therefore it obeys relativistic causality.

A typical wrong line of thinking is to say : if Alice can predict with certainty some results by Bob (perfect correlations, obtained when $a = b$), then either Bob's result is predetermined, or there are instantaneous actions at a distance. But this dilemma only applies in a classical framework, where particles' properties are defined in an absolute way, and where Bell's inequalities do apply.

A standard light-cone picture shows explicitly how contextual inference is used when the relevant information is locally available The corresponding predictions are inferences, not influences, so no "action or influence at a distance" is involved.

Light-cone picture of the EPR-Bohm-Bell scheme.



The photon pair λ and the measurement settings x and y are randomly generated in separated light cones. The earliest time for generating the results $a|x$ and $b|y$ are at the intersections of the light cones, when Alice's prediction $P^a(b|y\lambda, xa)$ about Bob's result, and Bob's prediction $P^b(a|x\lambda, yb)$ about Alice's result, become available. The resulting predictions can be effectively checked in the verification zone V, in the common future of all light cones.

From state vectors to modalities.

Summarizing, the violation of Bell's inequalities by QM theory and experiments can be explained by considering contextual inferences, and these in turn are ultimately allowed by the predictive incompleteness of the quantum state : getting actual probabilities for measurement results requires to specify a measurement context.

Note : similar ideas have been formulated by R. Balian et al, by considering that ψ provides only mathematical q-probabilities, whereas ψ completed by the specification of the measurement context provides true probabilities for mutually exclusive events.

To make physical sense of the QM formalism, one needs

a state (vector) $|\psi_n\rangle$ AND

an observable (operator) $\sum_k a_k |\psi_k\rangle\langle\psi_k|$ with $|\psi_n\rangle \in \{|\psi_k\rangle\}$

Both of them taken together define a physical modality.

Completing ψ ?

• If ψ is not complete, does it tell anything concrete by itself ? It does, because it indicates a set of contexts, corresponding to all the observables including ψ as an eigenvector, where the associated measurement result (eigenvalue) is predictable with certainty.

• In recent papers we have introduced a framework which makes a careful distinction between the usual ψ without a context, and the physical state within a context, called a **modality**. In this langage ψ is associated with an equivalence class of modalities, called an **extravalence class** : whereas the modalities are complete, because they are properties of a system within a context, ψ is not, because the context is missing by construction.

• This gives a nice outcome to the Einstein-Bohr debate, by confirming the incompleteness of ψ, and by telling how to complete it : one should add the context - that actually fits with the "very conditions for making actual predictions" required by Bohr's answer.

Consequences for (non)locality

The implications of the (in)completeness of QM are discussed in more details in other articles, but a few comments are in order :

— Contextual inferences correspond to what is usually called "quantum non locality", but they are not related to locality in a relativistic sense, but to the specifically quantum condition that requires to attribute physical properties to systems within contexts.

— We explained how (PC) can be violated by a non-deterministic theory, without any conflict with causality. On the other hand, deterministic theories do agree with (PC), and therefore must violate (EL) to be compatible with the violation of Bell's inequalities ; an example of such a theory is Bohmian mechanics.

Consequences for (in)completeness.

— We argue that ψ is predictively incomplete, but not that QM is incomplete in the sense of being erroneous. It can be completed by reintroducing the context "by hand" (as done in textbook QM), or in a more formal way by using algebraic methods.

— In an algebraic framework QM remains "universal", but there is no universal unitary evolution any more, so both systems and contexts can be managed, in agreement with empirical evidence ; see e.g. P. Grangier, "Completing the quantum formalism in a contextually objective framework", arxiv :2003.03121, Found. Phys. 51, 76 (2021).

— All these arguments are consistent with the CSM approach, as already presented in many talks and articles.

CSM construction : physical description.

- In quantum mechanics a physical object is a system within a context

- **System : subpart of the physical world**, ideally isolated from the outside.
- **Context : outside from the system**, the context is described classically and changing it involves continuously varying parameters (e.g. rotating a polarizer).
- **Modalities : well defined physical properties, attributed to a given system within a given context. Modalities are certain and repeatable once defined, as long as the system and the context are not changed.**

- **Fundamental postulate (contextual quantization) :** the **number of mutually exclusive modalities** for a given system in any given context has a **discrete fixed value** N, depending on the system but not on the context.

- **Major consequence :** in general the connection between modalities associated with different contexts **must be probabilistic**, otherwise there is a contradiction with the contextual quantization postulate.

- **Specific quantum feature** : some modalities may be connected with certainty through different contexts, this is called **extracontextuality.**

CSM construction : mathematical description.

- Consequences of the physical definitions :

1 : the changes between contexts form a continuous group (composition law : associative, neutral element, inverse, in general non commutative)

2 : all the modalities connected with certainty form an equivalence class, called extravalence class (relation between modalities : reflexive symmetric transitive)

- Mathematical postulate (obtained by induction, not deduction) :

A rank-1 N×N projector is associated with an extravalence class, and N mutually orthogonal projectors are associated with a context.

(or equivalently due to the spectral theorem : a CSCO is associated with a context).

- Recovering the usual QM formalism (obtained by deduction from the above)

From Uhlhorn's theorem : unitary transformations between contexts.

From Gleason's theorem : Born's rule.

- Caution : A projector $| \psi \rangle\langle \psi |$ does not define a modality but an extravalence class, and similarly a density matrix ρ without specified context has many possible decompositions in terms of probability distribution over modalities. **This is a consequence of the predictive incompleteness of ψ, as seen above.** How to complete ψ ? See next slide.

CSM construction : universality and completeness.

* Composite systems are described using tensor products as usual.

* **Contexts = infinite tensor product ?** Taking this limit breaks unitarity, and leads to sectorization in type III algebra (see : von Neumann 1939, "On infinite direct products").

* **Using a sectorized global algebra :** tensor product between two vN algebra **type I non commutative for the system ⊗ type III commutative for the context.** Globally all is type III commutative, and this provides **a complete description corresponding to the modalities**, and not to the usual ψ describing an extravalence class : ok. **The algebra is universal, but there is no universal wavefunction.**

* **Major point : there is no need to specify all details for the context** (this is not possible : there are « infinitely many » details), it is enough to label the different sectors and this is just what is provided by the classical description of the context.

* This description applies to any (isolated) system within a context, so it is also complete since it fully specifies a modality. It is also universal in the sense that it describes **anything, but not everything (it is a ToA, not a ToE).**

Complete picture in the algebraic approach

Generic landscape : system and context

System
Type I (countable basis)
Unitary equivalence
Superpositions

Cut

Context (unbounded)
Type III (uncountable basis)
Loss of unitary equivalence
Sectorization + updating

Generic measurement

from Gleason / Born

coupling **updating**

$$|\psi_i\rangle\langle\psi_i| \otimes \rho_i^{(C_1)} \Rightarrow \sum_j p_j \, |\phi_j\rangle\langle\phi_j| \otimes \rho_j^{(C_2)} \Rightarrow |\phi_k\rangle\langle\phi_k| \otimes \rho_k^{(C_2)}$$

Isolated system, modality in context C_1

Measurement in context C_2, system coupled to unbounded context => sectorisation

Isolated system, new modality in context C_2

Almost as usual but…

* $|\, \psi \,\rangle$ comes with an extravalence class, not a state
* the modality belongs to both a system and a context
* the cut has acquired a mathematical status !

Conclusion 1 : coming back to Bell's options

(2) There are influences going faster than light => Violation of « elementary locality » without faster than light signalling.

Strong tension with relativistic causality => to be avoided

or not : Bohmian mechanics makes this choice.

(3) The quantities a and b are not independent variable => Violation of « free choice », there are no random independant events.

Full predetermination, no actual randomness => to be avoided

or not : G. t'Hooft makes this choice.

(4) The whole analysis can be ignored, that's quantum mechanics => Violation of « predictive completeness », cannot be formulated in a classical or deterministic framework, but makes sense for QM.

Needs randomness and contextuality, => to be chosen !

then ψ has to be completed by specifying the context.

Completing ψ « from above » by specifying the context seems to be a valid option, but what about physical realism ?

Conclusion 2 : making a choice

What about realism ? Let's define **Physical Realism** as :
The purpose of physics is to study entities of the natural world, existing independently from any particular observer's perception, and obeying universal and intelligible rules.

Our choice : in QM we can keep Physical Realism, Free Choice, Elementary Locality, but then the description given by $|\psi\rangle$ is predictively incomplete.

How to complete $|\psi\rangle$? Not « from below » with hidden variables, but « from above » by specifying the context, in order to get a full probability law over a set of mutually exclusive results (modalities).

Completing ψ « from above » by specifying the context is a valid option for physical realism, under the QM empirical constraints.

THANK YOU FOR YOUR ATTENTION !

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