Quantum Cognition with application to persuasion

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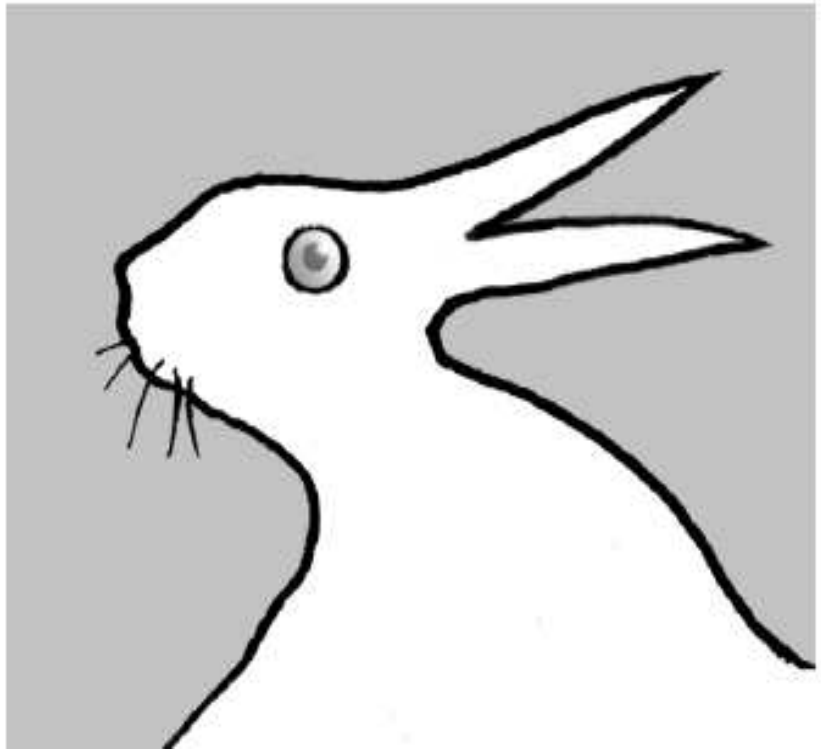
June 4 2018 Modélisation Quantique, ISC Paris

1. Quantum Cognition

- It is a common place that people are not capable of holding very complex pictures in mind : We consider one perspective and switch to another but face difficulties in combining them in a stable way.*
- This inability to seize reality in its full richness suggests that the cognitive process of learning may not (only)* look like a puzzle that is collected progressively.
- We propose that those difficulties be approached in terms of an incompatibility of perspectives for the human mind in a way that reminds of QM.

Ambiguous picture



Basic approach

1. Our knowledge about the world is always in terms of a representation, a mental picture i.e., knowledge is a mental construct, a psychic object.

2. We propose that cognitive limitations be modelled in terms of non-classical properties of this psychic object and we explore consequences for « learning ».

3. This can be viewed as a « reversal » of the standard QM approach: the « world » may very well be classical but the « mental picture of it» is not and this is what matters to decision-making, to how we live our lives.

Our approach cont.

- Most important element of our theory that we borrow from QM is Bohr complementarity of perspectives.
- Bohr complementarity or incompatibility of perspectives captures the cognitive difficulty in synthesizing information.
- As a consequence the process of investigation (learning) is not (fully) separable from the object of investigation (the (represented) world) , « we must rethink causality in our understanding of the world» (cf Dirac)

Our approach cont.

- Similar to the process of investigation in QM is the cognitive process i.e., the construction and evolution of the mental picture, we model it with 2 essentially different operations:

1. Information acquisition: we interact with the outside world by taking in « data » - we model it as *preparation operation*
2. Updating: an introspective operation by which we process information, we « translate » it in terms of a specific representation (our concerns), modelled as a *measurement operation.*

Preliminaries

Measurements: an interaction between a system (in our mind) and a device (a question). In our context an introspective operation on the mental picture. When 2 questions are incompatible, they modify the mental picture in a non-Bayesian way.

Pure and mixed mental pictures

Pure picture = maximal (but not complete) information state*

Mixed picture = incomplete info. In our context, the mental picture is mixed when

- prepared in a mixed state by acquisition of probabilistic info;
- an introspective msnt is initiated but not completed (decoherence);
- a superposed pure mental picture spontaneously decoheres.

The basic model

- The state of information/understanding of an agent, his *mental picture* *a pure state* $|\varphi\rangle \in \mathbb{H}$ or, more generally, a density operator

$$\rho = \sum_i \tau_i |\varphi_i\rangle\langle\varphi_i|, \quad \tau_i \geq 0, \quad \sum_j \tau_i = 1$$

- A *perspective (CSCO) R,* a question or observable i.e., an operator. The current picture in terms of any *R*

$$\rho = \sum_i \theta_i |r_i\rangle\langle r_i|, \ \sum_j \theta_i = 1.$$

- The agent has a *preferred perspective* R* related to some concern (intentionality)

No convergence of QL-learning

(1) Bayesian learning converges to the true state (s.t) A classically minded agent ends up learning the whole truth.

(2) with QC a maximal info state in some R is a superposed state in R* whenever R and R* are incompatible. With QC the mental picture never agregates all info into a complete information state instead the mental picture (cognitive state) jumps around for ever with the new info that comes.

$\Rightarrow$ Classical cognition and quantum cognition differ fundamentally

The Object of QL learning and knowledge about the world

As in QM, we cannot learn about the (represented) world directly and without affecting (the picture of) it: no first hand objectivisation feasible

As in QM a a major implication to this matter of fact is that

1. the proper object of learning about the world becomes the cognitive process: second hand objectivisation (cf Heseinberg)
2. We need a model of the mind (QC) that allows making predictions of the impact of new info and the outcome of introspective measurements - it is an invariant*. How different perspectives are correlated to each other should be the object of research.

Take-away message

- We start out with the postulate that our knowledge about the world is in term of a representation of it i.e., a mental construct.
- When acknowledging cognitive limitations in terms of QL-incompatibility of perspectives, this fact turns out to have far reaching consequences:

1. There can exist no unique truthful representation: we are faced with multiplicity of the true pictures and an irreducible uncertainty;
2. Any representation (even max info) depends of the path of learning: contextuality of the (represented) world
3. The proper object of knowledge is not (only) the outside world but the cognitive process (double objectivisation);
4. We have genuine creative freedom in the construction of the world we live in.

2. Quantum Persuasion

- Choice under uncertainty

-The Persuasion problem

- An illustrative example from political decision-making

- An Experimental Test

Choice under uncertainty:Lotteries

- A classical VNM lottery:
  A lottery l over X is defined by a collection of prize $(x_1, \ldots, x_r)$ and a prob vector $(p_1, \ldots, p_r)$. A measurement (a draw of a roulette) is performed and gives outcome $(1, \ldots, r)$.

Under well-known conditions the utility of l to our DM is given by

$$U(l) = \sum_i p_i u(x_i)$$

- A « horse lottery » AA is a function from S (set of states) to the set of lotteries over a given set of prizes. The DM assigns probabilities to possible outcomes. A Msnt is performed and depending on the outcome the corresponding lottery is played and a prize is obtained.

Quantum lotteries

- Also a bet on the outcome of a measurement. The difference is that the states and the events are described in the Hilbert Space Model.
- The HSM allows for measurements that cannot be performed simultaneously, they are *incompatible*. Some msnts do not commute i.e.,the state of the system changes with the msnt. An expression of this is that there does **not exist one single finest partition**.

Ex. 1. A bet on the outcome of the msnt of the spin of an electron along two different axes

Ex. 2. Behavioral: A bet on the realization of some events *evaluated subjectively:* Quantum Cognition.

Quantum Cognition

1. Our knowledge about the world is always in terms of a **representation**, a mental picture i.e., knowledge/information is a mental construct, **a psychic object**.
2. In face of complex phenomena we consider one perspective and **switch** to another but face **difficulties in combining them in a stable way**.*
3. Quantum Cognition proposes that those difficulties be approached in terms of an **incompatibility of perspectives** for the human mind in a way that reminds of QM.
4. More precisely, this incompatibility is modeled in terms of **Bohr complementarity of perspectives** (properties) of the psychic objects.

Elements of quantum decision theory

A *belief operator* B is a Hermitian non-negative operator with trace equal to 1.

In Physics B is the state of the system it allows predicting the (probabilistic) outcome of any measurement A : $Tr(AB)$.* In our context B is interpreted as the cognitive state wrt to a (represented object).

$\mathbf{Tr}(PB)$ is the subjective probability for event P when the cognitive state is B.

Theorem 1'.

Let $\preceq$ be a preference relation that satisfies A1-A5. Then there exist a utility function u and a unique belief operator B such that the preferences are represented by $U(\sigma) = Tr(Sh(\sigma)D)$.

Dynamic consistency

Suppose that a DM with ex-ante preferences given by $\succsim$, observes event P : her revealed preferences should change but how?

We show that from behavioral principles (axioms) we arrive at

Theorem 3. *Assume that* $\mathbf{Tr}(PBP) > 0$*. Then Axiom CU holds if and only if the belief operator is updated into* $D_P = PDP/Tr(BP)$.

- If P commutes with each P_i, then the updating rule is Bayes rule.
- If not, we obtain a generalization of Bayes (von Neumann and Luder's Postulate).
- Dynamic consistency does not entail the "sure thing principle" in its dynamic form $\Rightarrow$ new take on e.g. Persuasion.

Persuasion

- There are two players called respectively Receiver and Sender. Receiver chooses on action with uncertain consequences. To assess the value of the different actions Receiver uses her beliefs about the state of the world.
- Receiver's action also has consequences for Sender. Therefore Sender may try to persuade Receiver so she chooses an action favorable to him.
- For that purpose Sender's selects some information structure (or measurement) that generates new information (or signal) about the relevant uncertainty.

How can Sender persuade Receiver to select the action he wants?

Persuading a MP to vote No

The story is about the MP's (Receiver) decision to support or not a law that introduces a state of emergency to combat terrorism. An civil righ activist (Sender) wants her to vote down the law.

Let H be a 2-dim Hilbert space with basis (e_1, e_2)

$P_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $P_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$. The MP (Receiver) choose between Yes and No depending on how she subjectively evaluate the threat. Her utility:

$N = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$ $Y = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.

If our MP has belief about the level of threat given by $B = \begin{pmatrix} a & b \\ b & 1-a \end{pmatrix}$. The expected utility of the actions

$$EU(NO) = \mathbf{Tr}(NB) = 3a - 2$$
$$EU(YES) = \mathbf{Tr}(YB) = 1 - 2a$$

Let's assume that our MP holds initial belief (prior) $B = \begin{pmatrix} 1/5 & 2/5 \\ 2/5 & 4/5 \end{pmatrix}$ with this belief MP votes Yes.

The activist receives utility 1 with a NO vote whatever the true level of threat and utility 0 otherwise. With MP's prior belief, he gets 0.

Can the activist change the MP's mind?

In the classical context Sender is constrained by Bayesian Plausibility: expected posteriors=priors not in the quantum context.

Theoretical results in the quantum context:

- with an infinite sequence of msnts Sender can move Receiver's belief wherever he wants.
- with a sequence of 2 measurements S can already do very well.
- Q-persuasion exploits incompatibility: distraction rather than relevant information has significant persuasion power

Exploiting Bohr complementarity of perspective:Distraction

Let us consider another perspective on the law which we call 'the quality of public decision-making' (cf:EU decision about cucumbers).

This property is measured by the following direct measurement

(Q_1, Q_2) $Q_1 = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$ and $Q_2 = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}$.

P and Q are properties of the system (the world) that are incompatible in the mind of Receiver.

Whether the posterior B' is equal to Q_1 or Q_2then with probability $\mathbf{Tr}(Q_1 P_1) = \mathbf{Tr}(Q_2 P_1) = 1/2$ we have

Sender $EU = 0.5$

Above we considered the case when the target cognitive state was taken to be P_1. But take as a target $T = \begin{pmatrix} 3/5 & \sqrt{6}/5 \\ \sqrt{6}/5 & 2/5 \end{pmatrix}$.

This target (as a belief state of our MP) also induces action NO.

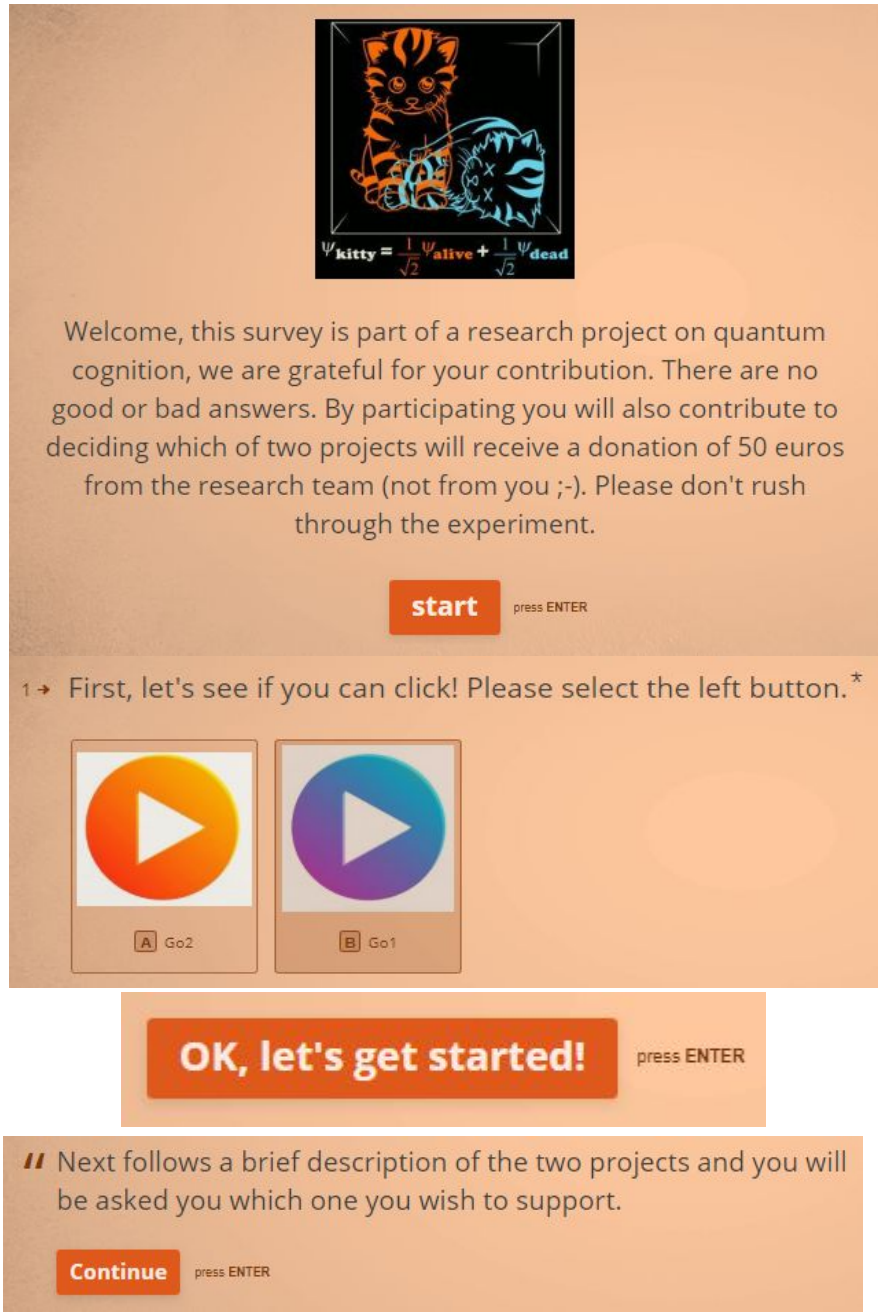
The probability NO for jump to 0.916!

The example illustrates how Sender can exploit the quantum indeterminacy of the cognitive state (expressed in the incompatibility of the two perspectives) to persuade our decision-maker.

By performing a measurement on an incompatible perspective, the cognitive state is modified, beliefs with respect to the threat are updated so that Receiver prefers to vote NO with probability close to 1.

Experimental test

1.
2. The participants were divided into three groups. Two treatment groups and a control group.



Tiger Forever : Tigers are illegally killed for their pelts and body parts used in traditional Asian medicines. They are also seen as threats to human communities. They suffer from large scale habitat loss due to human population growth and expansion. Tiger Forever was founded 2006 with the goal of reversing global tiger decline. It is active in 17 sites with Non-Governmental Organizations (NGOs) and government partners. The sites host about 2260 tigers or 70% of the total world's tiger population

Elephant crisis fund : A virulent wave of poaching is on-going with an elephant killed for its tusks every 15 minutes. The current population is estimated to around 700 000 elephants in the wild. Driving the killing is international ivory trade that thrives on poverty, corruption, and greed. But there is hope. The Elephant Crisis Fund closely linked to World Wildlife Fund (WWF) exists to encourage collaboration, and deliver rapid impact on the ground to stop the killing, the trafficking, and the demand for ivory.

Continue press ENTER

When considering donating money in support of a specific project to protect endangered species, different aspects may be relevant to your choice. Let us know what counts most to you

Continue press ENTER

1 → Which one of the aspects below is most important to you ?*

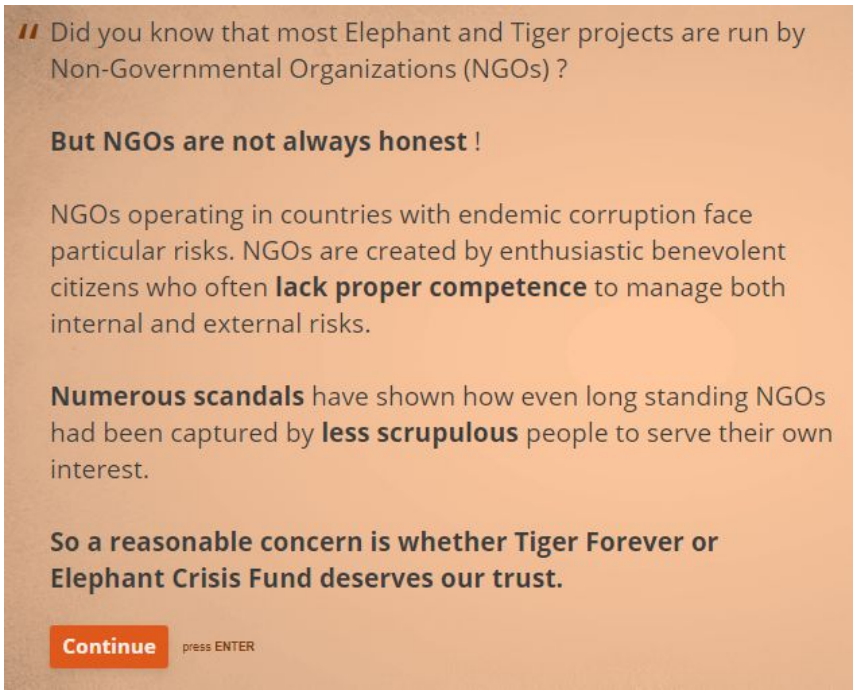
Urgency of the cause: among the many important issues in today's world, does the cause you consider belong to those that deserve urgent action ?

Honesty of the organization to which you donate: do you trust the organization managing the project to be reliable, i.e. do you trust the money will be used as advertised rather than diverted ?

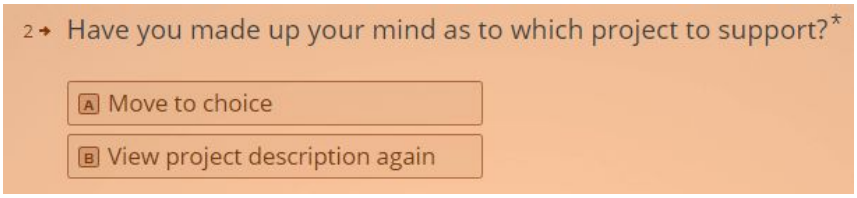
A Urgency

B Honesty

→ The Honesty extra-information



The Urgency extra-information



Results

The number of observations that were retained was 900. We conducted a number of probit regressions performed with Stata.

The first set of results establishes that incompatible information has a statistically significant* impact on the final choice which tends to significance **.

There is no impact of compatible information in any of those regressions.

None of the population variables had any impact on the final decision.

Interpretation

These results show that incompatible information that is "distraction" had a significant global impact on the final choice by inducing some extent of switch as compared to both the control group and the compatible information group.

These results are consistent with the predictions of the quantum persuasion model and contradict the predictions of the Bayesian model with respect to the impact of incompatible information.

Moreover the fact that general compatible information had no impact also supports the view that it is not merely "information" that affects the choice because the person is slightly "upset". Instead it is when information induces a change in perspective that something happens.