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Titre:

Applications de la relativité d'échelle à la mécanique quantique et à la turbulence hydrodynamique

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Résumé :

- La relativité d'échelle présuppose un espace-temps de nature fractale.
- On rappelera la forme de la dérivée covariante dans un tel espace-temps qui contient un opérateur de type Schrodinger ce qui permet notamment de fonder la mécanique quantique dans l'espace des positions. (x,t) fractal
- Tandis que pour la turbulence (hydrodynamique) on obtient l' équivalent (au niveau macroscopique) d'une équation de Schrodinger cette fois dans l'espace des vitesses. Matière fractale

Plan

- 1) some « definitions » of fractals objects, functions and of fractal/stochastic derivatives
- 2) some properties of Scale Relativity (SR)
- 3) SR applied to standard quantum mechanics ('Brownian' in x)
- 3a) short glimpse on QM 3b) results in SR
- 4) SR applied to hydrodynamic turbulence (Brownian in v)
- 4a) short glimpse on turbulence 4b) results in SR for Navier-Stokes equation (with SDE's)

- 5) new recent applications of scale relativity:
- - for rotating turbulence in geophysics
- -for turbulent free jet and shear flows
- -for turbulent boundary layers

- 6) provisory conclusions and perspectives

general « philosophy » here/maths

- If we have an underlying stochastic motion (ex uniform everywhere Brownian motion like in basic QM) then we can (prescription) replace the usual d/dt derivative by a suitable generalized derivative which has the stucture of a Schrodinger operator (using Ito calculus) to get a suitable « stochastic » derivative.
- Here no demonstrations are shown but just hypotheses/results and the search of their validation within real experimental data.

1) Fractal objects or functions

- Property : object structured at various scales (zoom)
- Curve or surface formed by deterministic or by stochastic rules
- Definition of fractality here :as explicit scale dependence
- Example : non differentiability +continuity ->fractality (theorem LN)
- $L(\varepsilon) \rightarrow \infty$ if $\varepsilon \to 0$
- Non stochastic fractals (ex: self similar ones with scale invariance, or not self similar),
- a) by iteration of functions like Cantor set, von Koch snow curve and so on
- b) by a recurrence relation between zooms like Mandelbrot –Julia set, Lyapounov fractal
- c) random factals like fractal landscapes in the sense of a probability to get a given structure at each zoom

Transformation $\varepsilon = dx$: simplest example $dL/d\ln\varepsilon = \varepsilon dL/d\varepsilon = cte$

$$\to L = L_0(1+(\lambda/\varepsilon)^{(Df-1)}) \qquad (0)$$

Df fractal dimension , then L is an explicit function of λ and of $dx = \varepsilon$ as a power law (in this case) :
f(x,dx) here is divergent in scale for $\varepsilon \to 0$, CP: $/\lambda$ transition scale fractal/non fractal … (see drawings
example : $\mathrm{Ln}(L/L_0) = g(\mathrm{Ln}(\lambda/\varepsilon))$….)

(which may correspond to stochastic/determinist motion transition)

- HERE : Random walk of Markov type, taken by analogy with Feynman path integral : non differentiable paths with Df=2

$dX = dx$ (differentiable part) $+d\zeta$ (non differentiable/stochastic part)

- $d\zeta$ in $dx^{(1/Df)}$, or δX in $\delta t^{(1/Df)}$

Transition fractal-non fractal: ex for $L(\varepsilon) = L0(1+(\lambda/\varepsilon)^{\tau f})$

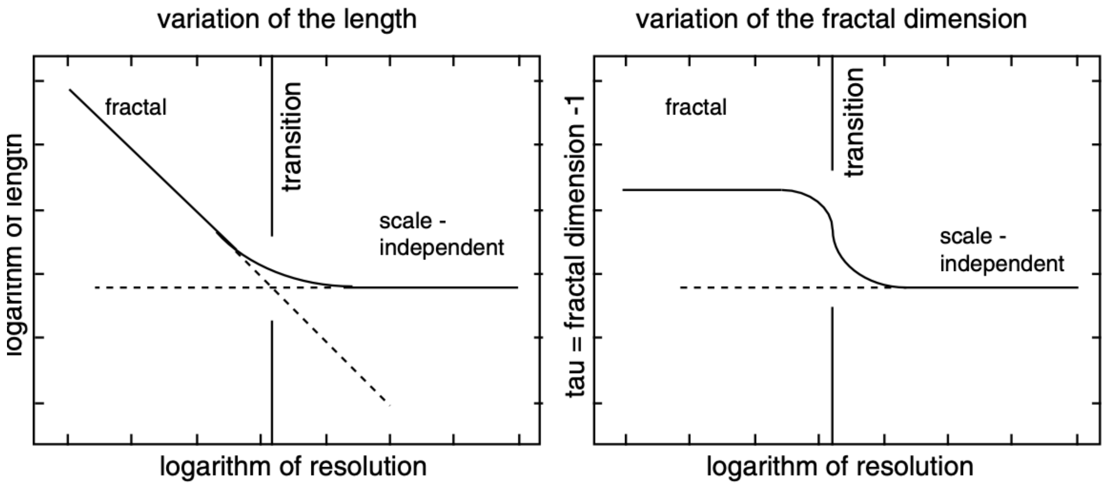


Fig. 4.1: **Fractal length and fractal dimension: self-similar fractal.** The two figures show the scale dependence of the length of a fractal curve (Eq. 4.8) and of its effective fractal dimension (Eq. 4.19) in the case of "inertial" or "Galilean-like" scale laws, which are solutions of a simple first order scale-differential equation. Toward the small scale one obtains a scale-invariant law with constant fractal dimension, while the explicit scale-dependence is lost at scales larger than some transition scale λ.

1) Fractals (2)

- stochastic "fractal" (also of class c above)

More general than (0) : $L(s, \varepsilon) = L0(s)(1+\zeta(s)((\lambda/\varepsilon)^{Df(s)-1}))$ with a stochastic process ζ –> stochastic fractal

- Stochastic process in usual sense of evolution (cc or discrete) of a random variable
- like typically diffusion cases : Brownian motion, Mandelbrot (fBm) , or more general (SDE's):
  $dx=f(x,t)dt + dBt$, $(dBt)^2=\sigma dt$ (1-2) CP $=\sigma=cte$
- For ex: if x(t,dt) is itself « stochastic fractal » as a coordinate then if put into a function F :

F(x,t)->F(x(t,dt),t): F becomes fractal itself externally through its fractal coordinate x , but in the GC :F could be both intrinsically and extrinsically fractal : for example F(x(t,dt),t,dt) …

- **Fractal « derivatives » (LN , ref book 1993)**
- if $f(x,\varepsilon)$ is a fractal function , then generically :

its "derivative" writes (at fixed ε) :

$df/dx=h(x)(1+y(x,\varepsilon)(\lambda/\varepsilon)^{\delta})$ and it diverges if ε->0 , then if ε varies y is also a function of ε

- and also $dy(x,\varepsilon)/dx = y(x,\varepsilon)(\lambda/\varepsilon)$ is itself scale divergent if ε->0

1) Fractals (3)

- Case of Brownian coordinates like with (1-2)
- ->Ito Lemma for fractal extrinsic derivative :
- Dh(x(t,dt),t)/dt=dh/dt+dh/dx f(x,t) + (σ/2) d^2h/dx^2

A Laplacian term appears here at second order in x to get a term of 1st order in dt (because the Brownian scales here as $dt^{1/2}$)

- For fractal functions f(t,dt) we can define 2 derivatives (which may coincide or not) : d+ and d- as :
- *d+f/dt =(f(t+dt,dt)-f(t,dt))/dt*
- *d-f/dt=(f(t,dt)—f(t-dt,dt))/dt*
- (dt is here a (physical) resolution and not a differential element in general)
- This possible dedoubling of derivative is specific of a fractal function at each point.
- The dedoubling of v (=dx/dt) or of a (=dv/dt) leads to use complex variables and we shall get a Schrodinger Kernel of the kind : i()d/dt+( ) d^2/dx^2 (see later)
- Main application here will be to QM and turbulence
- (Also link with multifractal functions : in the sense of having local fractal dimensions exponents but varying with the scale , like in turbulence…)

Stochastic derivative and embedding (1) (for D_f=2)

- Using Ito lemna+ complex variable one can build a « covariant » derivative d'/dt
- D diffusion coefficient, Vrond complex velocity
- with (d'/dt) to be applied next to Vrond itself

$$\frac{d'}{dt} = \frac{\partial}{\partial t} + \mathcal{V}.\nabla - i\mathcal{D}\Delta$$

Stochastic derivative (2)

- maths : if X (lies into a set Λ^1 of diffusion processes) solves a SDE such as : $dX(t)=b(t,X(t))dt+\sigma(t,X(t))dW(t)$
- With $X(0)=X_0$, W(.) Brownian motion on $I(0,1)xR^d$ and σ' s are Borel measurable functions (satisfying with b function suitable bounding conditions)
- AIM: to contruct an operator of « derivation D such that DX=dX/dt =usual derivative if X is a deterministic process
- D_+ and D_- operators left and right (as seen with d_+ and d_- above)
- Complex operator $D_k=(D_++D_-)/2 +ik(D_+-D_-)/2$, k=1 or -1
- Then for f(t,x) in $C^{1,2}(I \times R^d)$ and X in Λ^1 :
- $D_kf(t,X(t))=(dt(f)+D_kX(t).grad(f)+(i/2)a_{ij}d^2_{ij}f)(t,X(t))$
- $a_{ij}=(\sigma\sigma^*)_{ij}$,CP: σ=cte (diffusion coefficient) $a_{ij}d^2_{ij}f$-> $\sigma\ ^2d^2_{ii}f$ (Laplacian)

Stochastic derivative (3)

- Schrodinger operator : L and NL
- For Q=(X in $L^2/D_k{}^2(X)$=-gradU(X(t))) Newton case
- $\psi(t,x)=\exp(R/K+iS/K)(t,x)$, K>0 cte
- From d'/dt, Vrond(ψ)$=-2iK\, d_x \mathrm{Ln}(\psi)$ (comes from v in grad(S)/m in mechanics) then ψ obeys the following ENLS:
- ENLS: $(iKdt\, \psi + K(K-\sigma^2)/2\, (d_x\, \psi)^2/\,\psi +$
- $....(\sigma^2/2)\, d^2_{xx}\psi = U\psi$
- CP : K= σ^2 : ENLS->LNS :
- $i\,\sigma^2 dt\, \psi + (\sigma^4/2)\, d^2_{xx}\, \psi = U\, \psi$

message principal de la partie I

- Fractal au sens explicitement dépendance d’échelle (ou de résolution)
- On décompose les coordonnées ‘fractales’ en 2 parties : déterministe et stochastique par ex /abscisse curviligne s
- $dXi = dxi + d\xi i = vi\ ds + \zeta i\ ((2Di)\ ds)^{1/2}$ pour Df=2
- Avec ζi processus stochastique tel que $< \zeta i >=0$ et $< \zeta i\ ^2>=0$

- Une fonction f(x(t),t,ts) a pour dérivée (pour Df=2)
- $df/dt= \partial f /\partial t +v\ \partial f /dX\ +D\ \partial^2 f/\partial\ X^2$

- Pour $<dX/dt>=v$ et $<dX^2/dt>=2D\ dt$

2) A short introduction to SR (1), main ideas

- Theory developped by Laurent Nottale (Luth) over 40 years
- Hypothesis :In general the whole space-time itself is supposed to be fractal
- A system will depend at which scale(s) it is oberved

(with a lot of applications such as in astrophysics and cosmology)

-'Brownian' seed in x space : for δx in $\delta t^{1/2}$->we get the foundation of Quantum Mechanics (QM) space-time itself

-Brownian seed in v space for δv in $\delta t^{1/2}$->the turbulent (non differentiable) velocity (pressure etc…) creates its own fractality in v space : multiscale stochastic process (as in MF approach of turbulence). Matter itself

2) A short introduction to SR (2) , main ideas

- Principle of relativity of scales
- The theory of SR and fractal space-time initial target was to obtain a foundation of quantum mechanics (QM) on first principles. This has been achieved by deriving all the QM postulates from the only principle of relativity, once it is generalized to scale transformations of the coordinate system.
- The theory is based on the relaxation of the hypothesis of space-time differentiability.
- One can prove that a continuous but non-differentiable space (more generally space-time) is fractal, under the general meaning of an explicit scale-dependence of coordinates on the resolution scale and of their divergence when the resolution interval tends to zero

This theorem is the basis for the scale-relativity method of dealing with non-differentiability: the various physical functions become fractal functions explicitly scale-dependent, $f = f(t,\tau)$, so that one can define a derivative in t at any given scale τ, $f'(t,\tau) = \partial t(f(t,\tau))$, even though it no longer exists at the limit $t \rightarrow 0$.

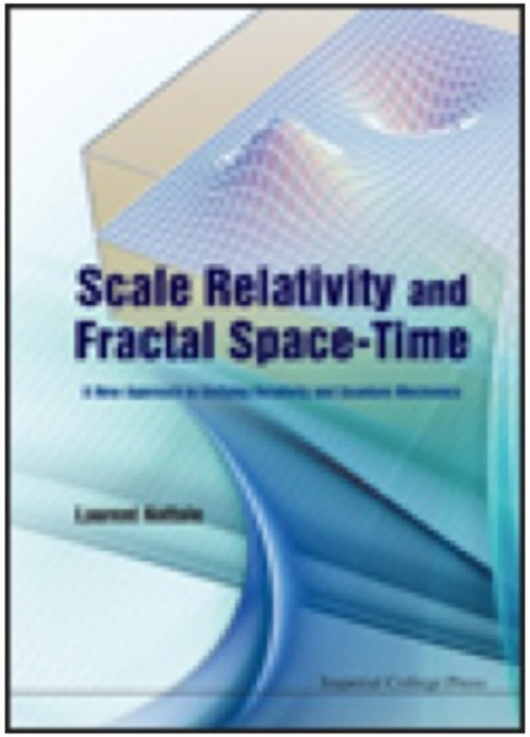
2) A short introduction to SR (3)

- In other words, while the standard differentiation method keeps only the limit lim(f) for dt→0 and thus fails when this limit does not exist, the scale-relativity method keeps all the history of what happens when dt → 0. We lose nothing in this way, since when the limit exists it is included in the description, while when it does not exist we still keep a description tool which is physically effective, i.e. at finite resolution scales (while the null resolution interval is a mathematical concept which has no physical meaning).
- d^/dt (covariant derivative) accounts for the geometric effects of a fractal space, we can writte the equation of motion in such a space as a geodesics equation, i.e. under the form of Galilean free inertial motion, d^V*/dt = 0. This equation can be generalized to d^V/dt = −∇ϕ in the presence of an exterior potential and then integrated under the form of a Schrödinger-type equation. (With d^/dt and V* to be defined below)

2) Scale relativity (LN)

- (Book announcement (LN 2011)) fractal geometry of space time itself (x,t)/not only of functions or objects
- Principles , fractality, explicit scale or resolution dependence of functions such as v(x,dx,t,dt) , introduction of complex derivatives with d+,d-, with zoom on dt,dx…

  start with generic case : dx=vdt+dW, deterministic +stochastic part
- Covariant derivative (as see)
- Link with quantum mechanics (derivation of QM eqs. Schrodinger, but also of Pauli +spin, and Dirac in relativistic case …algebra doubling, Clifford algebras) ,
- quantum mechanics as special fractal (Brownian like) motion with $D_F=2$. But Markov process without memory. (From SR derivation of possible more general eqs. for $D_F \neq 2$)
- New features : transition classical-quantic (in turbulent case) like classical/quantum in QM or fractal/non fractal
- Emergence of a stochastic potential $Q=-2D^2\Delta_{xx}(\mathbf{P}^{1/2})/\ \mathbf{P}^{1/2}$
- (+applications…)
- Possible macro-quantic states/ cases , since “the coefficient ” of diffusion D in the Schrodinger is no more hbar/2m (microscopic case) ; examples structure formation in astrophysics and cosmology, quantified Kepler laws, Hartree eqs….

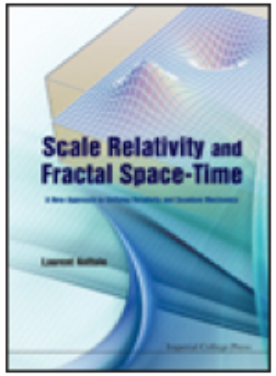


Book about Scale Relativity (ICP 2011)

SCALE RELATIVITY AND FRACTAL SPACE-TIME

A New Approach to Unifying Relativity and Quantum Mechanics

by **Laurent Nottale** *(Paris-Meudon Observatory, France)*



This book provides a comprehensive survey of the development of the theory of scale relativity and fractal space-time. It suggests an original solution to the disunified nature of the classical-quantum transition in physical systems, enabling the basis of quantum mechanics on the principle of relativity, provided this principle is extended to scale transformations of the reference system. In the framework of such a newly generalized relativity theory (including position, orientation, motion and now scale transformations), the fundamental laws of physics may be given a general form that unifies and thus goes beyond the classical and quantum regimes taken separately. A related concern of this book is the geometry of space-time, which is described as being fractal and nondifferentiable. It collects and organizes theoretical developments and applications in many fields, including physics, mathematics, astrophysics, cosmology and life sciences.

2) Scale relativity (SR) , Properties

- Some specific features (see later on)
- Letting the resolution going to zero but not taking the limit…(in physics : finite resolution of measurements)
- 3 Items are useful/necessary in the general case to implement the SR :

a) Existence of an infinity (or very large number) of geodesics (trajectories)

Vi(s) ->a scale-dependent velocity field Vi[Xi(s, ds), s, ds].

b) Irreversibility in the change dt->-dt :

->doubling at each point of the (time) derivatives

2) Scale relativity (SR) , Properties…

- X+’ (s,ds) = (X(s+ds,ds)−X(s,ds)/ds,
- X−’ (s,ds) = (X(s,ds)−X(s−ds,ds)/ds
- X’+,- tranforms into each other under relection if ds->-ds
- But not coincide
- c)The geodesics are « fractals » (with a given Df)
- V+[x(s, ds), s, ds] = v+[x(s), s] + w+[x(s, ds), s, ds]
- V−[x(s, ds), s, ds] = v−[x(s), s] + w−[x(s, ds), s, ds]
- Have a differentiable and a fractal part
- ->construction of the Covariant derivative //Ito calculus for stochastic processes

Covariant derivative in SR to account for fractal geometry

- $d_\pm x = v_\pm \, dt$, $d\xi_\pm = \zeta_\pm (2D \, dt)^{1/2}$
- d^/dt = (d+/dt + d−/dt)/2 − i/2(d+/dt - d−/dt)

- V* = d^/dt (x (t)) = V − i U = (v + v −)/2− i /2(v + − v −)
- by combination of d+f/dt et df-/ dt (Ito Lemna of part 1) we get :

- (Vrond=V*)

$$\frac{\hat{d}}{dt} = \frac{\partial}{\partial t} + \mathcal{V}.\nabla - i\mathcal{D}\Delta$$

2) SR (2)

- Link to Renormalization Group (scale group) in QFT
- Also derivation of a FP equation equivalent to the Schrodinger eq. by relating v and Ψ by a specific relation (also see below)
- Link with hyp of quantum foam of (x,t) of J.A Wheeler+recently quantum 2D gravity with Df=2

Message for part II…/SR

3) Reminder of previous general results of SR applied to QM

3a) Some reminders about QM

(Brownian motion of x (in x space))

3b) derivation of QM by the SR

3a) Standard Quantum Mechanics (QM) (1)

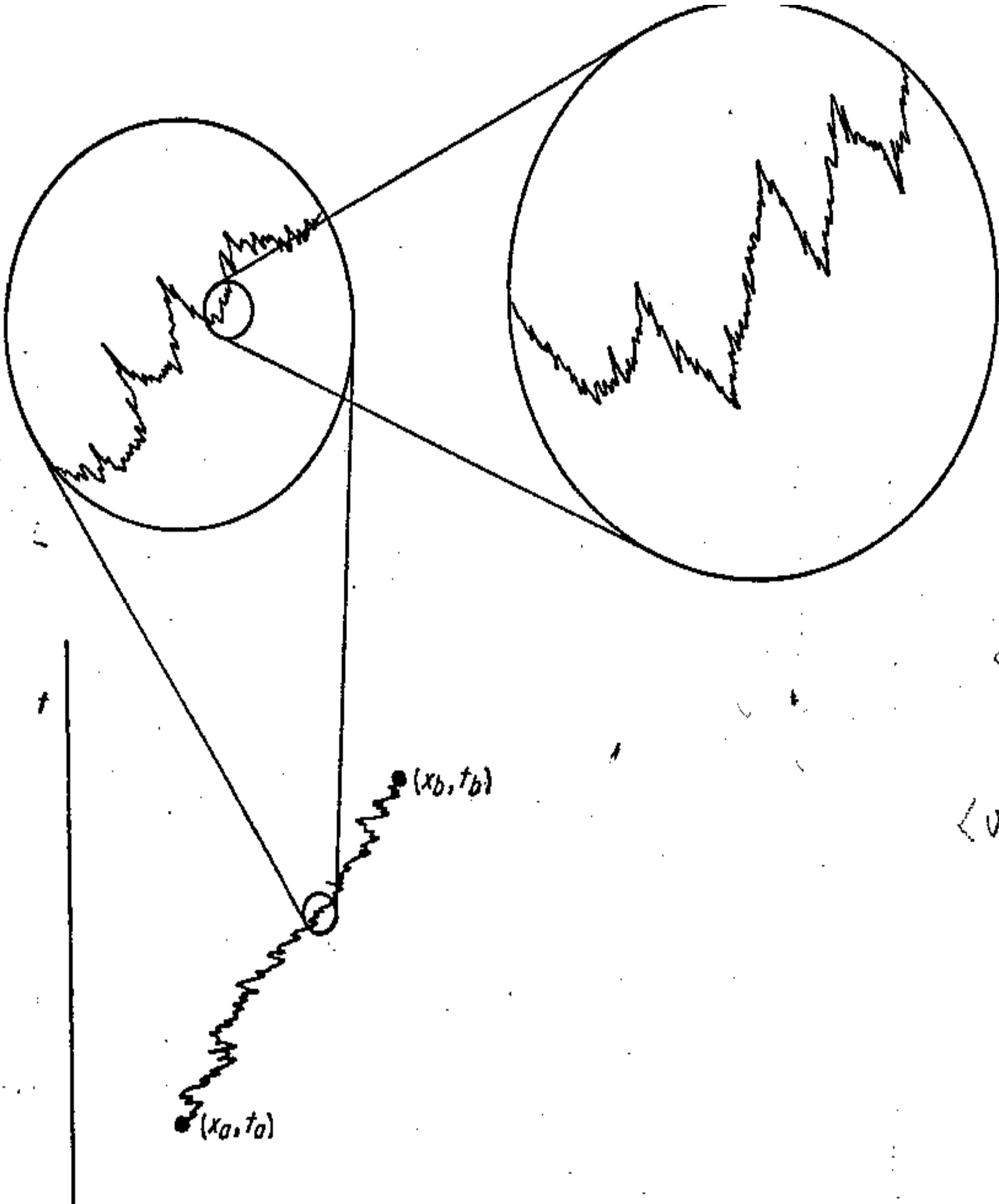
- Derivation of the Schrodinger equation in “usual” physics by principle of correspondence from Hamilton-Jacobi(HJ), Action…(see derivation if needed+NL terms in HJ)
- Schrodinger equation , wave function squared integrable in relevant Hilbert space
- Probabilistic interpretation of particle presence in real space
- Example of quantum “trajectory”, Feynman path integral

3a) Schrodinger equation

- Properties …linear ELS , ENLS…
- Example of “quantum paths”

->No real trajectory but Pty : P(x,t)=ΨΨ*

- See Feynman 1948 : 1^{st} microscopic theory of QM with micro Probabilities of modulus 1 ,since here Ψ in exp(iS)…(S action)
- Paths are highly irregular at fine scale, here in (x,t) plane , paths are non differentiable.


t
x
(x_b, t_b)
(x_a, t_a)

QM (2)

- p→(hbar/i) $grad_x$, E->(-hbar/i) dt…

- Poisson bracket (symplectic class M) becomes commutators and anti-commutators…(Lie algebra) , spin

- Heisenberg inequalities, De Broglie-Bohm wavelength…
- Remark: quantum pressure often dropped in usual derivation of Schrodinger eq. (scales in $hbar^2$)…
- “Natural link” between Schrodinger and NS equations by the Madelung transformation (possible generalization in the non zero vorticity case)
- Here natural geometrical “analogy” between the bundle of geodesics in QM and in fluid turbulence (fractal like trajectories) even if in different spaces or space-time (QM : dx^2 in dt, while for dv^2 in turbulence) . The spaces themselves are fractal here at fine scale.
- ->fractal geometry…

3b) RE->QM : FRACTAL SPACE-TIME (x,t) –> QUANTUM MECHANICS

Covariant derivative operator (adapted by Ito calculus for the case here dx^2 in dt)

$$\frac{\acute{d}}{dt} = \frac{\partial}{\partial t} + \mathcal{V}.\nabla - i\mathcal{D}\Delta$$

Fundamental equation of dynamics

$$\frac{\acute{d}\,\mathcal{V}}{dt} = -\nabla\phi$$

Change of variables (S = complex action) and integration

$$\mathcal{S} = \int_{t_1}^{t_2} \mathcal{L}(x, \mathcal{V}, t)dt \qquad \psi = e^{i\mathcal{S}/2m\mathcal{D}}$$

Generalized Schrödinger equation

$$\mathcal{D}^2\Delta\psi + i\mathcal{D}\frac{\partial\psi}{\partial t} - \frac{1}{2}\phi\,\psi = 0$$

Refs LN…NB: More general :for a force NOT deriving from a potential : we get a Schrodinger Kernel SK = iDdt+$D^2\Delta$, to be inverted on the force source term.

FRACTAL SPACE-TIME –> QUANTUM MECHANICS (2)

Covariant derivative operator

$$\frac{\acute{d}}{dt} = \frac{\partial}{\partial t} + \mathcal{V}.\nabla - i\mathcal{D}\Delta$$

Fundamental equation of dynamics

$$\frac{\acute{d}\,\mathcal{V}}{dt} = -\nabla\phi$$

Specific used relation : here Vrond=V-iU,

Vrond=-2iDgrad$_x$(Ln(Ψ(x,t))) from the usual : v=grad(S)/(m) , S action

D here =diffusion coefficient with aspecific expression according to the system under study, in usual QM one recovers : D=hbar/2m

CP Newton eq with potential Φ

$$m\frac{\acute{d}}{dt}\mathcal{V} = -\nabla\Phi$$

$$\mathcal{V} = \nabla\mathcal{S}/m$$

$$\psi = e^{i\mathcal{S}/\mathcal{S}_0}$$

$$i\mathcal{S}_0\frac{\acute{d}}{dt}(\nabla\ln\psi) = \nabla\Phi$$

$$\mathcal{V} = -i\,\frac{\mathcal{S}_0}{m}\,\nabla(\ln\psi)$$

$$\nabla\Phi = i\mathcal{S}_0\left(\frac{\partial}{\partial t} + \mathcal{V}.\nabla - i\mathcal{D}\Delta\right)(\nabla\ln\psi)$$

$$\nabla\Phi = i\mathcal{S}_0\left[\frac{\partial}{\partial t}\nabla\ln\psi - i\left\{\frac{\mathcal{S}_0}{m}(\nabla\ln\psi.\nabla)(\nabla\ln\psi) + \mathcal{D}\Delta(\nabla\ln\psi)\right\}\right]$$

$$\frac{\acute{d}}{dt}\mathcal{V} = -2\mathcal{D}\nabla\left\{i\frac{\partial}{\partial t}\ln\psi + \mathcal{D}\frac{\Delta\psi}{\psi}\right\} = -\nabla\Phi/m$$

$$\mathcal{S}_0 = 2m\mathcal{D}$$

Schrödinger

$$\mathcal{D}^2\Delta\psi + i\mathcal{D}\frac{\partial}{\partial t}\psi - \frac{\Phi}{2m}\psi = 0$$

Ref. : LN, Fractal Space-Time and Microphysics, World Scientific 1993, chap. 5

Four (equivalent) representations

Geodesic (*U*,*V*)

$$\frac{\widehat{d}\mathcal{V}}{dt} = -\nabla\left(\frac{\phi}{m}\right)$$

Generalized Schrödinger (ψ) == ($P^{1/2}$,θ)

$$\mathcal{D}^2 \Delta \psi + i\mathcal{D}\frac{\partial}{\partial t}\psi = \frac{\phi}{2m}\psi$$

Euler + continuity (*P*, *V*): Q quantum potential, P=/ψ/[2]

$$m\,(\frac{\partial}{\partial t} + V\cdot\nabla)V = -\nabla(\phi + Q)$$

$$\frac{\partial P}{\partial t} + \mathrm{div}(PV) = 0$$

New potential (quantum) energy:

$$Q = -2m\mathcal{D}^2\frac{\Delta\sqrt{P}}{\sqrt{P}}$$

Diffusion (v_+, P_-) Fokker-Planck equation , v doubling in (v+ ,v-)

$$v_+ - v_- = 2\mathcal{D}\nabla \ln P$$

$$\frac{\partial P}{\partial t} + \mathrm{div}(Pv_+) = \mathcal{D}\Delta P$$

here SR in usual x space

- *dx=vdt+dW* , dW=ζ (2Ddt)$^{1/2}$
- dX=dx (differentiable part) +dW(non differentiable/stochastic part)
- dW in dx$^{(1/Df)}$, or dX in dt $^{(1/Df)}$
- Random walk of Markov type, taken by analogy with Feynman path integral : non differentiable paths with Df=2

- *Irreversibility : doubling of v vector v->(v_+,v_-)*
- *->Vrond=(v_+ + v_--)/2-i(v_--v-)/2=V-iU*
- *since d+f=(f(t+dt,dt)-f(t,dt))/dt differs from*
- *d-f=(f(t,dt)—f(t-dt,dt))/dt*
- *Vrond=-2iD gradx(ln(Ψ(x;t)) : potential part of velocity*

ITO like derivative : d'/dt=d/dt+Vrond.gradx-iDΔxx

-> equation of motion for fluid case : Navier-Stokes eq. (like Newton law for a single particule)
dv/dt=-grad(p) +ηΔv ,if x Brownian :
then (for x stochastic) replace d/dt by d'/ dt, and v by Vrond

Geodesics 1

- By constructing a scale-covariant derivative which accounts for the geometric effects of a fractal space, we have written the equation of motion in sbuch a space as a geodesics equation, i.e. under the form of Galilean free inertial motion, $d'V/dt = 0$. This equation can be generalized to the Newtonian form $d'V/dt = -\nabla\phi$ in the presence of an exterior potential (which can itself be the manifestation of an inner geometric property and then integrated under the form of a Schrödinger-type equation.
- The non-differentiability and fractality of coordinates implies at least three consequences :
  (1) The number of geodesics is infinite. Their description naturally becomes non-deterministic and probabilistic. The ensemble of these paths therefore constitutes a (virtual) fluid, which is characterized by its velocity field.
- (2) Each geodesic is itself fractal with fractal dimension $D_f = 2$ corresponding to the Markovian nature of motion in a fractal space.
- (3) The non-differentiability also implies a two-valuedness of the (scale-dependent) deriva- tive of the coordinates, (V_+, V_-),. Indeed, one needs one point to define a position, but two points to define a velocity, so that two definitions now exist (the second point being before or after the position point), which are no longer invariant under the reflexion transformation $|dt| \rightarrow -|dt|$ in the non-differentiable case .
- These three properties of motion in a fractal space lead to describing the velocity field of geodesics in terms of a complex fractal function $V_e = (V_+ + V_-)/2 - i(V_+ - V_-)/2$. The (+) and (−) velocity fields can themselves be decomposed in terms of a differentiable part $v_\pm$ and of a fractal (divergent) fluctuation of zero mean $w_\pm$, i.e., $V_\pm = v_\pm + w_\pm$. Therefore the same is true for the full complex velocity field, $V_e = V(x, y, z, t) + W(x, y, z, t, dt)$.

Geodesics 2

- The elementary displacements along these geodesics can be described in a stochastic way,
- $dX\pm = d\pm x + d\xi\pm$, with $d\pm x = v\pm\, dt$, $d\xi\pm = \zeta\pm \sqrt{2D}\, |dt|^{1/2}$
- Here $\zeta\pm$ represents a dimensionless stochastic variable such that $<\zeta\pm> = 0$ and
- $<\zeta\pm^2> = 1$. The parameter D characterizes the amplitude of fractal fluctuations.
- These various effects can be combined in terms of a total ("scale-covariant") derivative operator which generalizes to a fractal space the Euler derivative $\partial t + V.\nabla$, by adding two imaginary terms to it:
- $d\hat{}/dt = \partial t + V^* . \nabla - i D \Delta$, $V^*=U-i V$
- Newton's fundamental equation of dynamics becomes, in terms of this operator :
- $m\, d\hat{}\, V^*/dt = - \nabla \phi$
- In the absence of an exterior field ϕ, this is a geodesic equation (i.e., a free inertial Galilean-type equation),
- $D\hat{}V^*/dt=(\partial t +V^*.\nabla - iD\, \Delta)V^*=0$ (same V*)

Macroscopic Schro eq.

- The final step consists of making a change of variable in which one connects the velocity field V = V − i U to a complex function ψ = eiS/S0 (where S is the action, now complex because the velocity field is itself complex), according to the relation
- mV = −i S0 ∇ ln ψ
- This equation is but the standard relation between momentum and action P = ∇S, that provides a new expression (now exact) for the principle of correspondance.
- This relation plays a fundamental role in the scale-relativity theory, since it translate the geometric description (left hand side, in terms of velocity field of geodesics of the fractal space-time) into the standard QM algebraic description (right hand side, wave function)
- Thanks to this change of variables, the equation of motion can be integrated under the form of a Schr¨odinger equation generalized to a constant S0 which may be
- different from hbar :
- $(D^2\Delta+iD\partial t-\phi/2m)\ \psi=0$ where D is another expression for S0 according to the relation:
  S0 = 2mD
- (which is just another form for the Compton relation λ = hbar/mc in standard QM).
  By setting finally $\psi = P^{1/2} \times \exp(i\theta)$, with V = 2D∇θ, one can show that $P = |\psi|^2$
- gives the number density of virtual geodesics. This function becomes naturally a density of probability when the geodesics are manifested in terms of effective particles. The function ψ, being solution of the Schr¨odinger equation and subjected to the Born postulate and to the Compton relation, owns therefore all the properties of a wave function.

Trajectories in SR (bundles of geodesics) method for numerical computation

Given a local potential + symmetries+ BC

- → Schrodinger eq. --> solution Ψ-->
- solution pour Vrond = -2 i D gradx ln Ψ(x,t)
- → solution V or for v+,-> SDE for QM :

dx=v+(or V)dt+dW (Brownian coordinates)

- $dW=\zeta\,(2Ddt)^{1/2}$ with ζ stochastic such as $<\zeta>=0$ and $<\zeta^2>=1$
- Simulations allow to “reconstruct” the “true” quantum paths…

Provisory conclusion on the SR' status/message part III

- SR=Microscopic theory of QM relying on SDE's
- as seen : with Feynman also : microscopic theory of QM but with wave functions of Pty =1 (in exp(iS)) with action S=int(L dx)…L= Lagrangian

- The wave function is no more postulated in SR like in usual quantum mechanics (QM) but it is now built as a manifestation of the fluid of virtual geodesics, quite in the spirit of the Feymann path integral approach.

- At large scale other covariant derivative with effect of curvature included of space-time (x,t) in General Relativity

4) SR applied to hydrodynamic turbulence (Brownian seed in v)

- 4a) the state of the art in fluid turbulence, open pbs , intermittency, anomalous exponents…
- 4b) SR for Navier-Stokes equation (with SDE's)
- Next application here : to turbulence but now in v space (willing to reconstruct also the turbulent observed trajectories)

4a) on fluid turbulence

- Turbulence NS et Euler equations, still puzzling since nature of singularities if any are not yet known (other ongoing research with Euler-Leray and NS-Leray equations) . Reminder on turbulence spectrum (HIT).
- Multi-fractal behavior well observed in data but the theory is still at the level of phenomenology/dynamics
- Importance of signal analysis (but no examples here)
- Proposal for a new approach (a little bit less phenomenological?) based on a macro-quantic Schrodinger equation. (Link with NS-Euler by Madelung "like" transformation)

Some open pbs in fluid turbulence

- Intermittency : deviations to the Gaussian law for the distribution of random processes ex : large tails of the pdf(v) and pdf(a) , bursts of turbulence
- (here some answer may come from SR)

- Anomalous dissipation : in 3D ‘inertial dissipation’ is still not well understood

- Existence of (energy) cascades ? Hyp of K 41-K62 but if (quasi) singularities occur : no need of cascades to transport the energy through scales (collapse of singularities)

NS equations

- Equations NS 3D compressible or incompressible

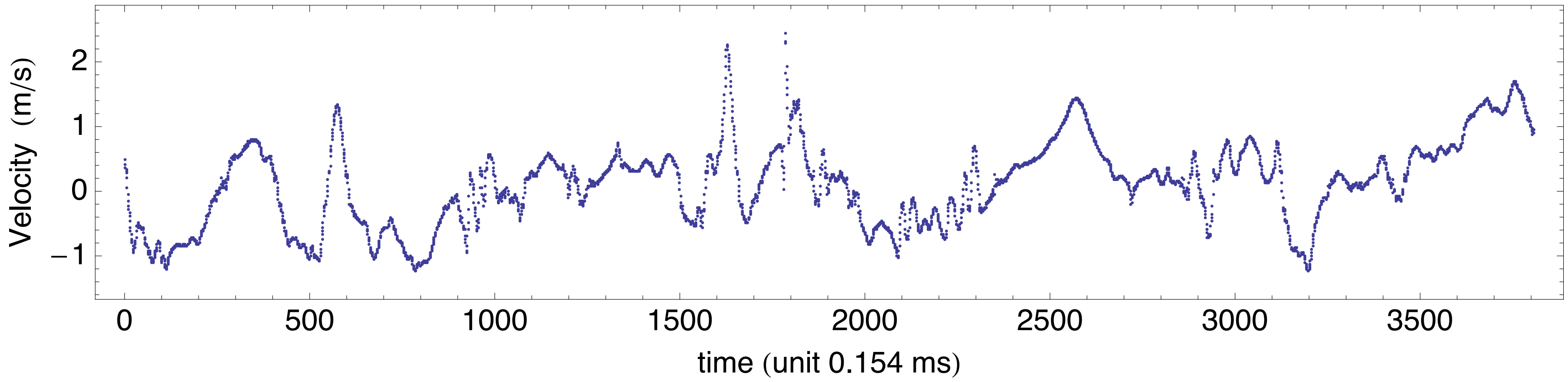
$$\partial_t \mathrm{v}(\mathrm{x},t) + \mathrm{v}.\nabla \mathrm{v} = -\nabla p / \rho + \nu \, \Delta_{\mathrm{xx}} \mathrm{v}(\mathrm{x},t)$$

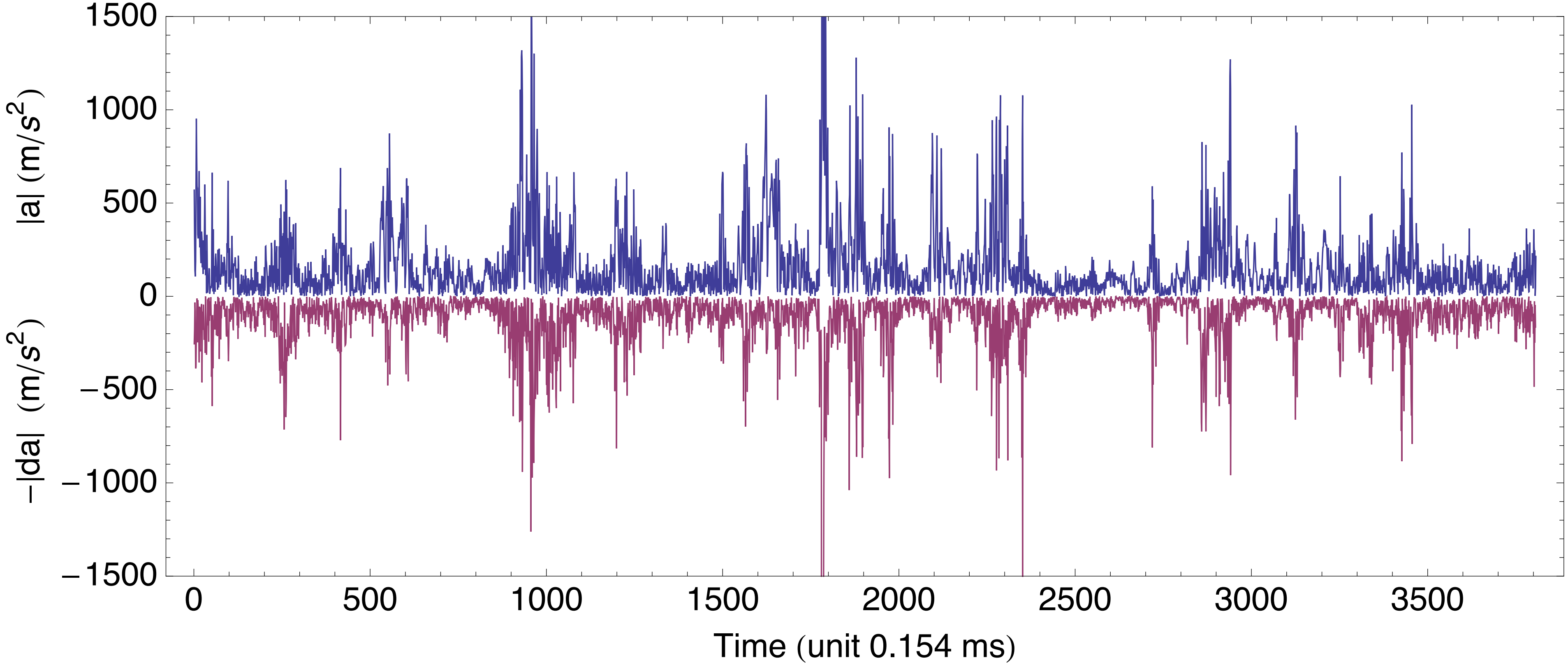
$$\partial_t \rho(\mathrm{x},t) + \nabla.(\rho \mathrm{v}) = 0$$

- Poisson equation for pressure p :
- incompr. case , but vorticity source NL term

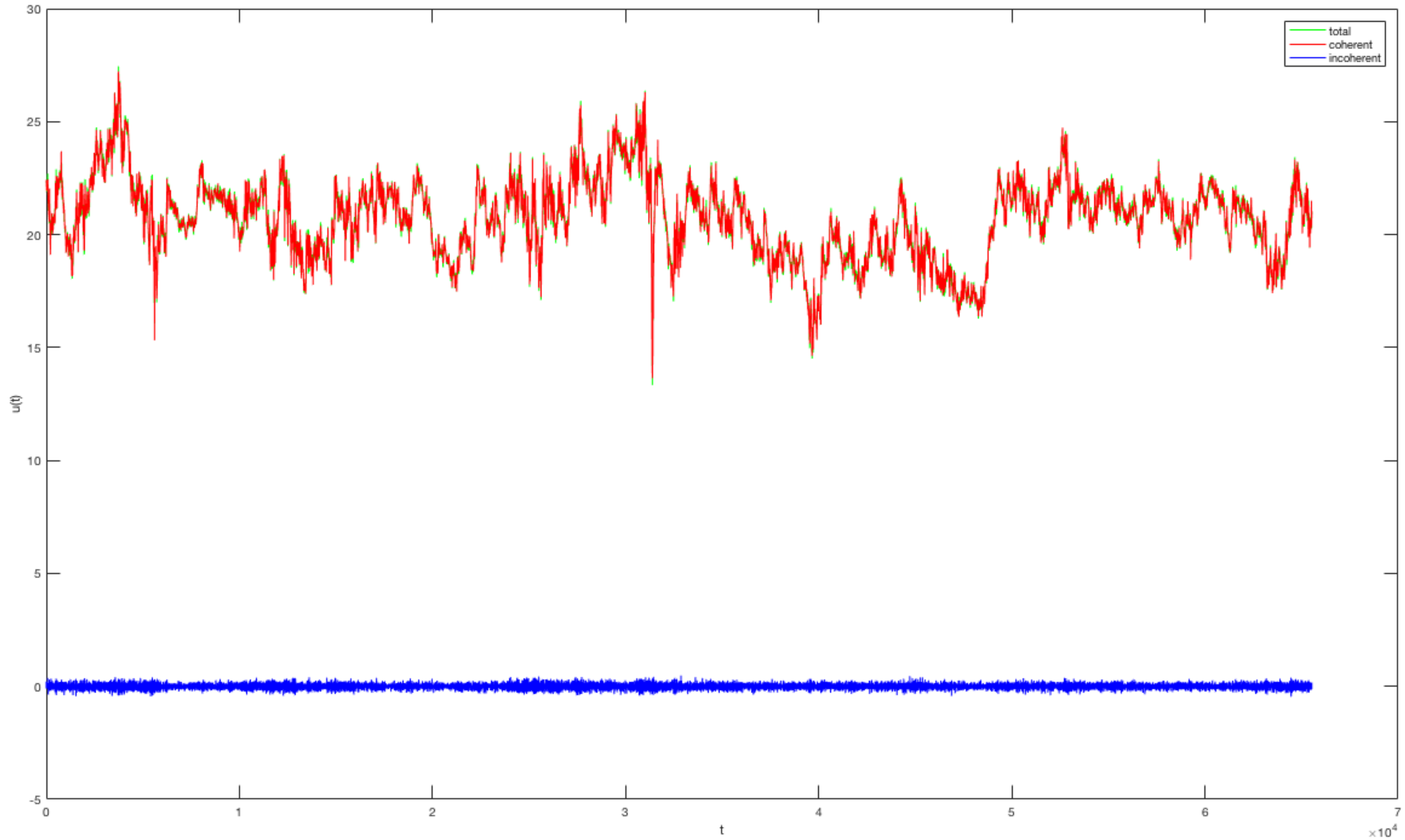
$$\Delta p = (\partial_{x_i} \mathrm{v}_j \partial_{x_j} \mathrm{v}_i) = S((\nabla \mathrm{v})^2 \rightarrow p = \Delta^{-1}(S)$$

Exs: Mordant data Lagrangian L , $v_L(t)$ +intermittency of $a_L(t)$



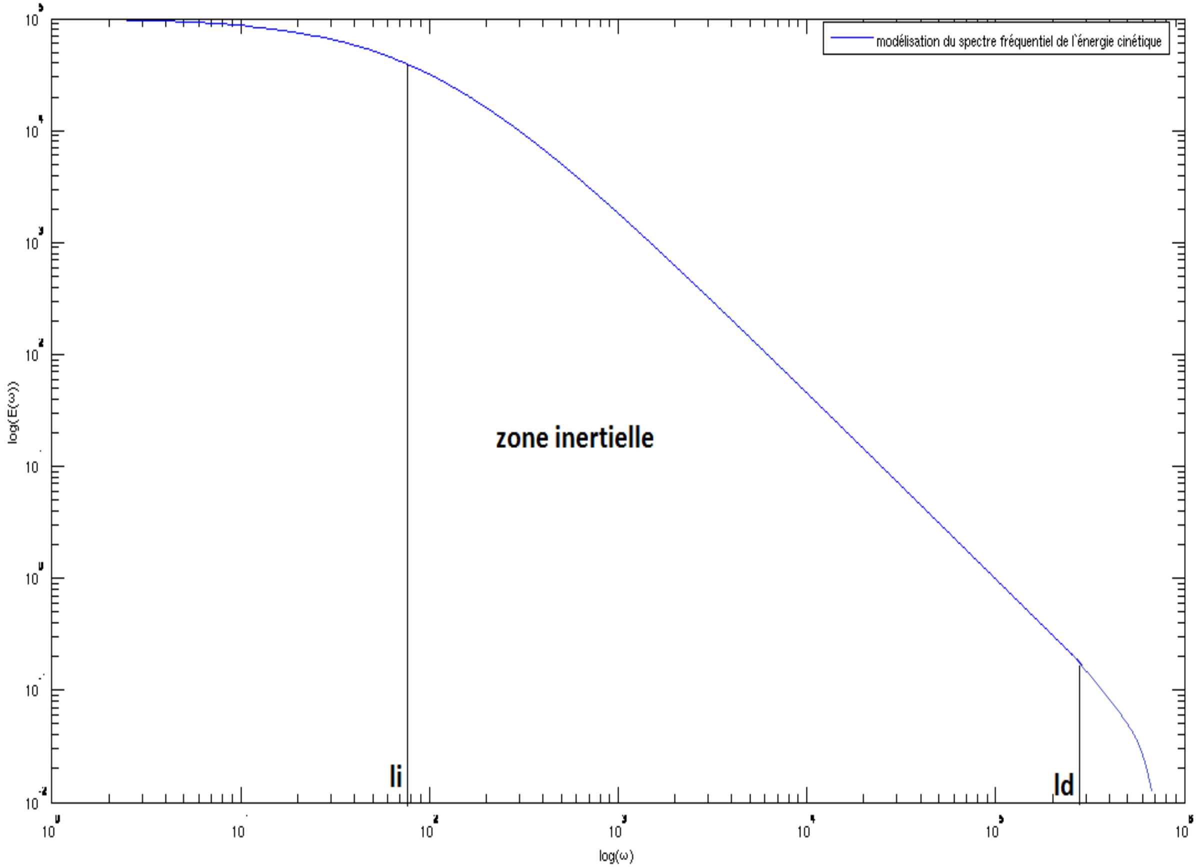


Ex : $v_E(t)$ data Eulerian E (Modane 1995)

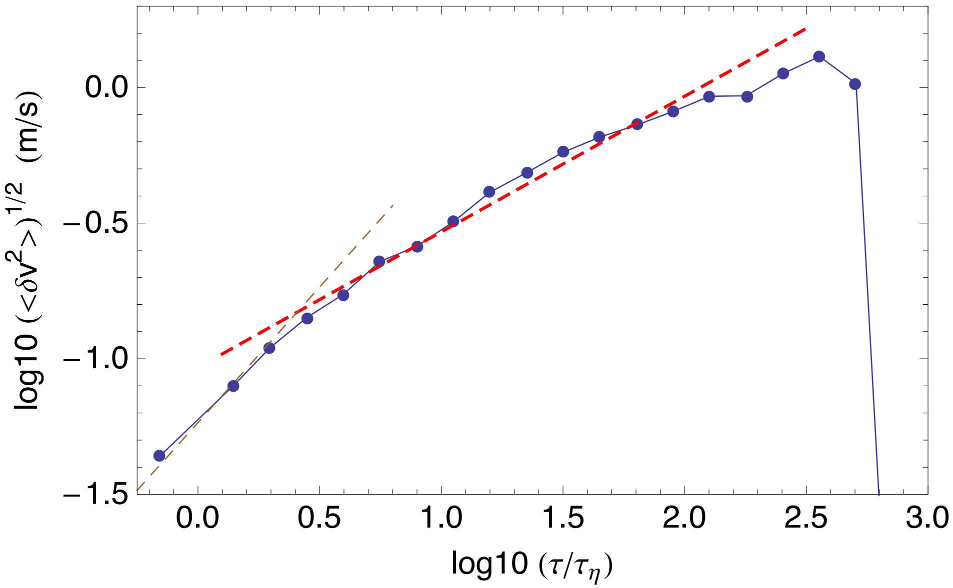


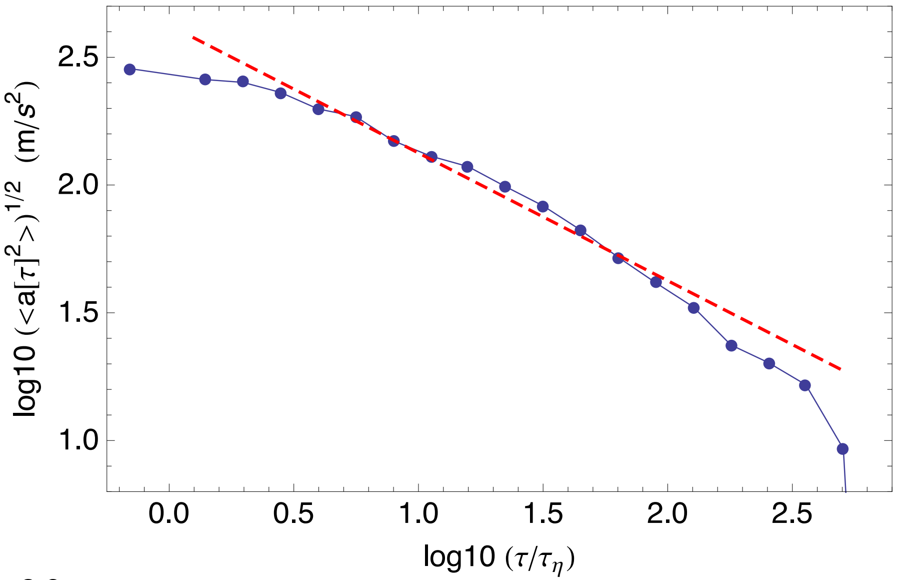
Modane spectrum $E_c(k)$ in $k^{-5/3}$ in E, in ω^{-2} in L , inertial and dissipative domains (THI case) like fractal/non fractal

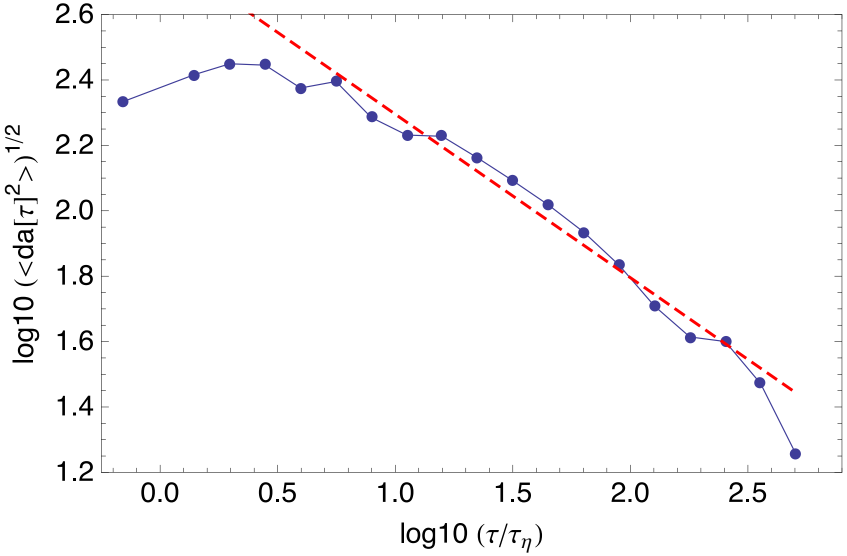
$l_\eta=(\nu^3/\varepsilon)^{1/4}$, $\tau_\eta=(\nu/\varepsilon)^{1/2}$ $\varepsilon=$rate of interscale energy transfer

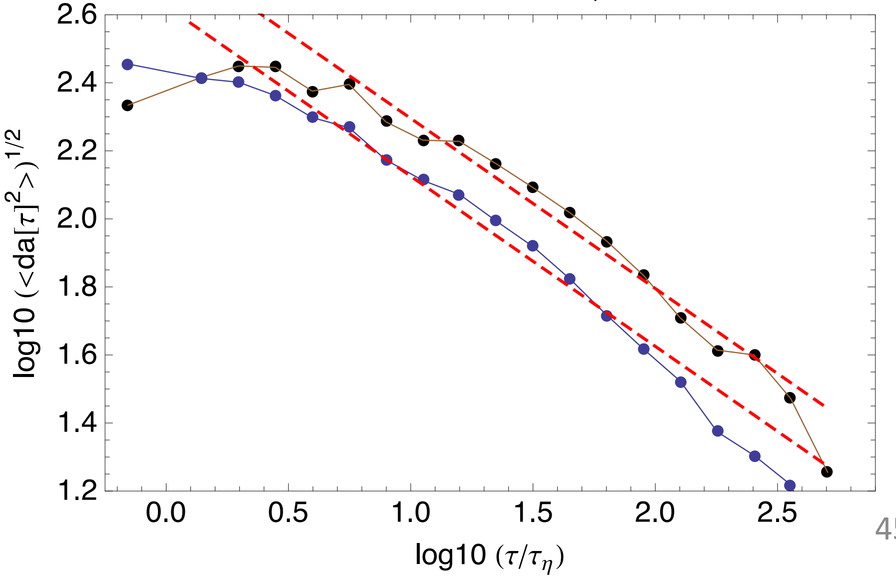


Mordant data L: man290501, Seg3398, scaling laws {$dv \approx dt^{1/2}$, $a \approx dt^{-1/2}$, $da \approx dt^{-1/2}$} (idem with E data with exponent 1/3 : dv in $dx^{1/3}$) Brownian in v

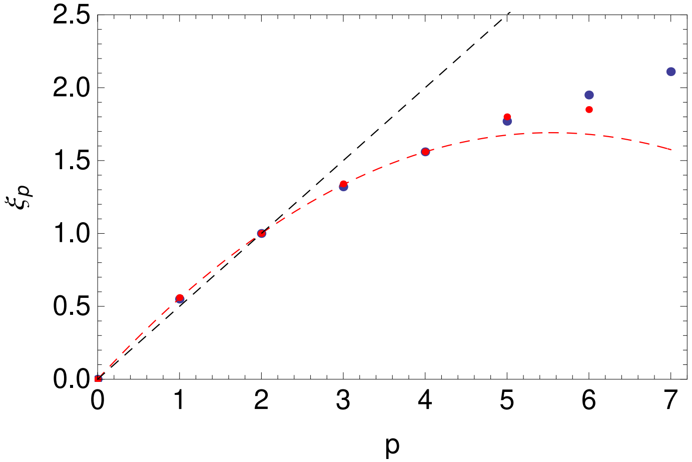








anomalous exponents in HIT on structure order v functions <δ$v_L^p(\tau)$> in $\tau^{\zeta_{p,L}}$



4b) Main results of SR in (HIT) turbulence

relativity methods in *velocity*-space to turbulence description

(réf : L. De Montera, 2013, **A theory of turbulence based on scale relativity,** arXiv:1303.3266)

- New *Schrödinger-like* form of motion equations (on the derivative of NS eqs.)
- Main consequence: existence of *a new* « divergent » *component of acceleration*
- Comparison to global experimental data *: aim to describe large non G tails* of the accelerations' PDF (predictions of PDF)
- Comparison to local experimental data : a new proposed mechanism for *intermittence* bursts
- (not shown in detail here : Scaling laws and link to related multifractal studies)…..

Main results (2)

- Prediction of intermittency (here for Lagrangian data), mecanism
- Identification of quiet zones (Gaussian), and burst zones
- Validation of the multifractal model used for the L data
- Specific data analysis (local minima…)
- Look for universality , quantum like oscillators as local potentials (demonstration +validation on data)

Here : **Lagrangian** approach

- Possible trajectories of a Lagrangian tracer *in velocity-space* (de Montera 2013):
- - chaotic -> stochastic description ;

Brownian motion but now in v space and ‹ $\delta v^2 \propto \delta t$ › ,
with fractal dimension D_f=2 (in v space)

. local irreversibility : symmetry breaking under time reflexion :

→two-valuedness o $\delta t \leftrightarrow -\delta t$

→3 conditions like in x but now in v space + add 4) newtonian regime and 5) sufficient range of fractal scales

Conditions for Schr¨odinger equation in v space

- The transformation and integration of the fundamental equation of dynamics into a Schr¨odinger equation in nondifferentiable and continuous (therefore fractal) geometry is a general mathematical result which does not depend on the nature of the variables. It just relies on the properties of the derivative of a fractal variable (in the new sense defined hereabove) and on its dynamics described by its second order derivative.

- In the case of turbulence, the position coordinates are, a priori, not fractal inside the relevant scale range, i.e. between the Kolmogorov dissipative scale and the integral scale. However, the velocity of fluid particles becomes fractal according to Komogorov K41 scaling law. This suggests applying the scale-relativity theory to turbulence in v-space, i.e. to the velocity v as coordinate and the acceleration a as its derivative.
- This application relies on the fact that the various conditions on which the derivation of a Schr¨odinger equation is based are satisfied in the inertial range of a turbulent fluid
(1) The number of possible trajectories of a fluid particle (or of a reliable Lagrangian tracer) is infinite, due to the highly chaotic nature of turbulence.
(2) The Kolmogorov K41 scaling $\delta v \sim \delta$ in the inertial range means that the trajectories in v-space are fractal of dimension 2. There is no strict non-differentiability, since this range is lowerly limited by the Kolmogorov time-scale $\tau\eta$. But its effects are manifest through the recognized scale-dependence and divergence of accelerations toward small scales, $a \sim \tau^{-1/2}$ down to $\tau = \tau\eta$.
(3) The non-differentiability also implies a two-valuedness of the (scale-dependent) deriva- tive of the v-coordinates, i.e. of accelerations ($A+$, $A-$), from which one constructs the complex variable $A = (A+ + A-)/2 - i\,(A+ - A-)/2$. We have shown that this behav- ior is manifest in turbulence data , in which one finds that the accelerations a and their increments da are of the same order of size, contrarily to the basic assumption of differential calculus, $da \ll a$.

- (4) The motion is Newtonian, i.e., the acceleration is proportional to the force applied (even though there are also non-Newtonian contributions like viscosity).
(5) The range of scales on which the effective fractal dimension is $DF = 2$ should be large enough: this condition is fulfilled for large enough Reynolds numbers, since this range is simply given by $TL/\tau\eta = R\lambda/2C0$, where $R\lambda = \sqrt{15Re}$ and $C0 \approx 4 - 7$.

Application to fluid turbulence :
v = is now the basic coordinate

$$dv = a\,dt + dW \qquad dW = \zeta\sqrt{2D_v dt}$$ (K41)

$$a \longrightarrow \{a_+, a_-\} \quad \rightarrow \mathcal{A} = \frac{a_+ + a_-}{2} - i\,\frac{a_+ - a_-}{2}$$

(Irreversibility -> doubling of acceleration vector)

$$\mathcal{A} = -2iD_v \nabla_v \ln \psi_v$$

(potential part of acceleration)

$$\frac{\widehat{d}}{dt} = \frac{\partial}{\partial t} + \mathcal{A}.\nabla_v - iD_v\Delta_v$$

(New form of total derivative)

→ Motion equation : Navier-Stokes → (ρ=1,if incompressible)

$$\frac{dv}{dt} = F = -\nabla p + \nu\Delta v$$

Derivative of NS: $\dfrac{da}{dt} = \dot{F}$ → $$\frac{\widehat{d}}{dt}\mathcal{A} = \dot{F}$$

Detail on equation of motion with force going from x to v **coordinates (x,v(x))->(v,a(v))**

We would like to write dF/dt as a function of v, t only ,but not always possible , to make the link with the force source term in the NS equation.

dF/dt=?-gradv(Φ(v,t)), not obvious even in the potential case. But validation is searched here directly in the data +demonstration later for the main relevant term in d(-grad(p)/dt).

(Add dem if useful for Φ(v,t) in $\Sigma_{i,j} a_{ij} v_i v_j$)

(In theory open pb for the link between E and L data for turbulence)

Schrödinger form of mean motion equation in special case : dF(v,t)/dt=-gradv(Φ(v,t))

$$D_v^2 \, \Delta_v \psi_v + i \, D_v \frac{\partial \psi_v}{\partial t} - \frac{\phi}{2} \psi_v = 0$$

$$P_v(v) = |\psi_v|^2$$

With dissipation -> Non-Linear Schrödinger , with a term in ψ ln ψ → **Quantized**

Back to stochastic initial description : ***A* is now known**

$$\psi_v = \sqrt{P_v} \times e^{i\theta} \qquad dv_+ = A_+ \, dt + dW$$

$$A_+ = A_U + A_V = D_v(\partial_v \ln P_v + 2\partial_v \theta)$$

Sub-special case : θ = cst (ex 1D . : harmonic oscillator, psi *real*) --> $A_+ = A_q$

$$A_q = D_v \, \partial_v \ln P_v$$

Refs. : L. Nottale, arXiv:1306.4311; Cells 2014, 1, 1-35; LN, *Space-Time Geometry and Quantum Events*, Chapter 5, Ed. I. Licata, Nova Science Pub., 2014.

Consequences of the Schrodinger macro-quantic like equation

- P(v) is the Pdf of v, to be validated with data
- P(v,a)? How to derive it ? Would give the correlations between v and a…
- Try with stochastic Langevin Lagrangian eqs. (but not really validated by the data yet)
- Expression of the coefficient D_v (see later on) simpler case $D_v=D_0=C_0(Re)\varepsilon$, ($\varepsilon=\langle a.v\rangle$) energy rate transfer through scales (in k_x or in k_v space) D_v=hbar,eff in v space

General Acceleration A /1D case

- more general case with full formula later on for A since in the case of a 2D (or 3D) oscillator (see for turbulence in rotation) the phase is no more constant /1D case . For Ψ=/Ψ/exp(iθ/(2D_v)),
- At 1D Aq just seen with Pv=/Ψ/^{2}
- at 2D : A+ and A- are now (2D) vectors

- A+=D_v gradv(ln(Pv))+(2Dv) grad(θ)
- A-=-D_v gradv(ln(Pv))+(2Dv) grad(θ)

now P(A+) and P(A-) to consider

- Determination of coefficient D_v (théo+ on data) and of the harmonic potential oscillator Φ(v) (théo+validation on data)

Interpretation

- Development of turbulence → non standard velocity differential element + irreversibility → two-valuedness of acceleration
- The turbulent material medium is fractal and locally nondifferentiable in v-space, acts on particles moving in this medium (Lagrangian tracers and fluid particles themselves) → ’ feedback loop ’
- The motion of these particles partly acquires « macro quantum-like » properties/Schrodinger equation

Refs. : L. Nottale, Fractal Space-Time and Microphysics, World Scientific 1993; L. Nottale, Scale relativity and fractal space-time, Imperial College Press 2011.

To find D_v : for ex from second order stochastic models (Lagrangian case/empirical /data) SDE (0)

Krasnoff and Peskin 1971, Sawford 1991, Pope2002

$$\frac{dv}{dt} = \beta_1 v + f. \qquad df/dt = \beta_2 f + dW_f/dt$$

Langevin damping in *v*

$$T_L = 2C_0 \frac{\sigma_v^2}{\varepsilon};$$

$$dv/dt = -v/T_L + dW_v/dt$$

$$da/dt = -a/T_a - a/T_a T_L + dW_a/dt$$

$$\sigma_a^2 = a_0 \frac{\varepsilon}{\tau_\eta}.$$

$$T_a = \frac{C_0}{2a_0} \tau_\eta,$$

Langevin damping in *a*

Harmonic oscillator in v-space

$$D_v = \frac{1}{2} C_0 \varepsilon = \frac{\sigma_v^2}{T_L}.$$

$$\omega_s^2 = \beta_1 \beta_2 = \frac{1}{T_a T_L} = \frac{\sigma_a^2}{\sigma_v^2},$$

$$D_a = \frac{\sigma_a^2}{T_a} = T_L \frac{\sigma_a^4}{\sigma_v^2}.$$

New theoretical predictions

- Velocity PDF : globally close to Gaussian (Mordant 2001, Voth et al 2002, etc.), but :
- Locally : strongly deviates from Gaussianity (Heppe 1998, etc.)
- Here: $P_v(v) = |\psi_v|^2$; psi > 0 and < 0 → *P_v* has
- minima at *P_v* = 0

$$A(v) = \mathcal{D}_v \frac{\nabla_v P_v}{P_v}.$$

→ Divergence of acceleration on minimas of Pv

→ **Predicts large tails of acceleration** $\hbar_v = 2D_v$

Tails of acceleration PDF : ***P*[*v*]=0**

- General case : near a zero Ψ= *g* $P_v \propto (v - v_1)^2$
- Corresponding acceleration (v_1 = 0):

$$P_v(v) = \left(\frac{v}{v_0}\right)^2 \qquad\qquad A(v) = 2\mathcal{D}_v/v$$

- Derived asymptotic PDF of acceleration P(a) from P(v) :

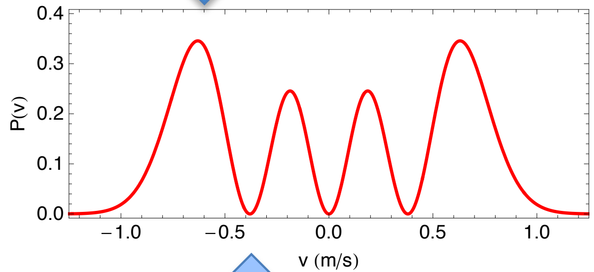
$$P_a(a) = \sum_k \frac{1}{|A'[V_k(a)]|} P_v[V_k(a)], \qquad P_a(a) = \frac{(2\mathcal{D}_v)^3}{v_0^2} \frac{1}{a^4}$$

Test of theoretical predictions with experimental data : acceleration PDF

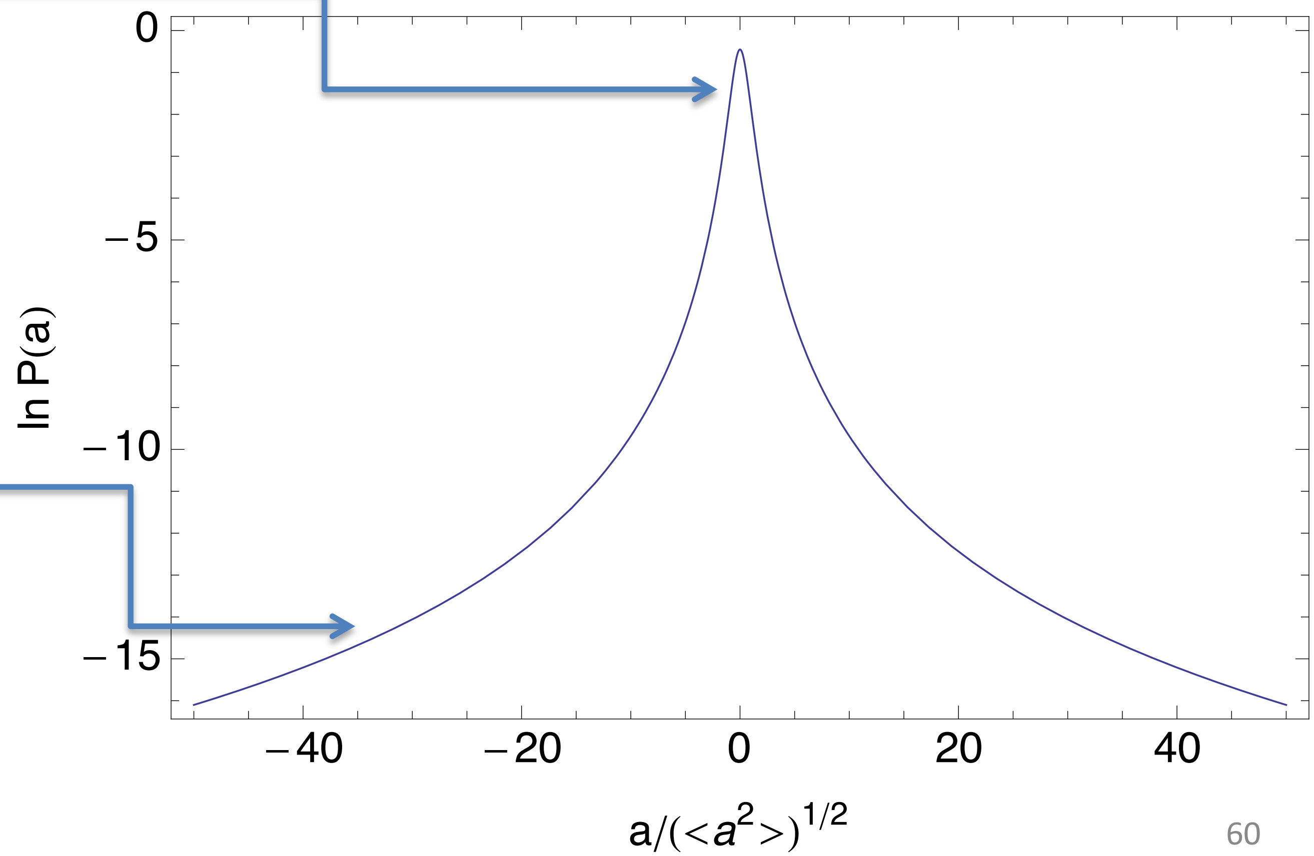
Examples: quantized harmonic oscillator,
≈ particle in a *v*-box, etc.. -->

$$P_a(a) = \frac{2}{\pi\sigma_a}\,\frac{1}{[1+(a/\sigma_a)^2]^2}.$$

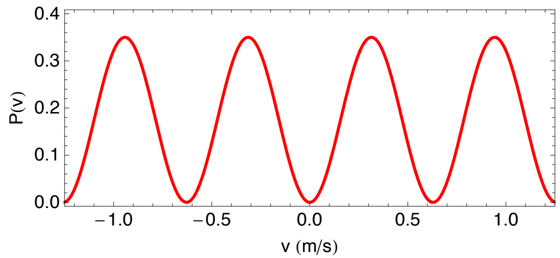
Peak of proba



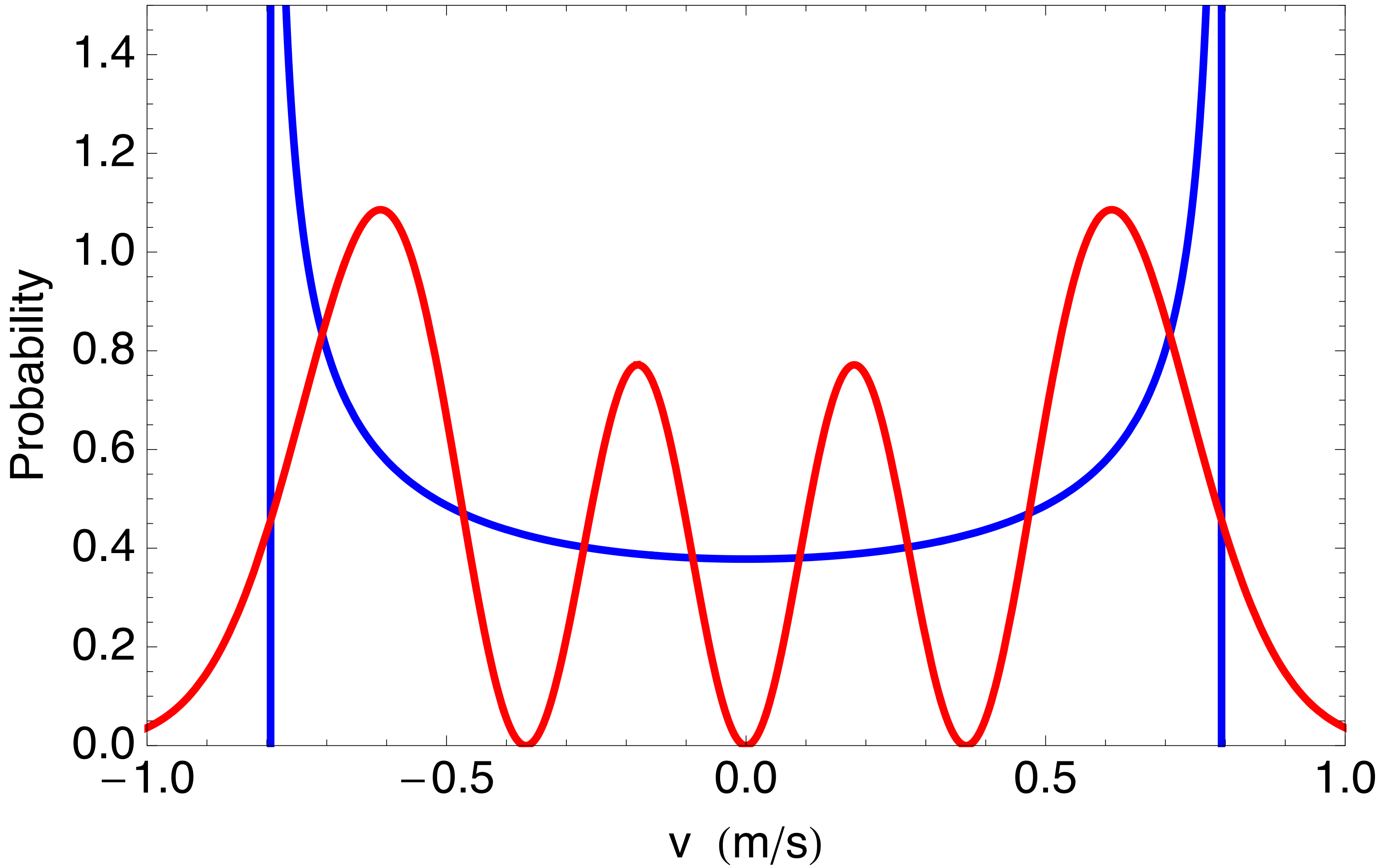
Tails of proba



v-Box :

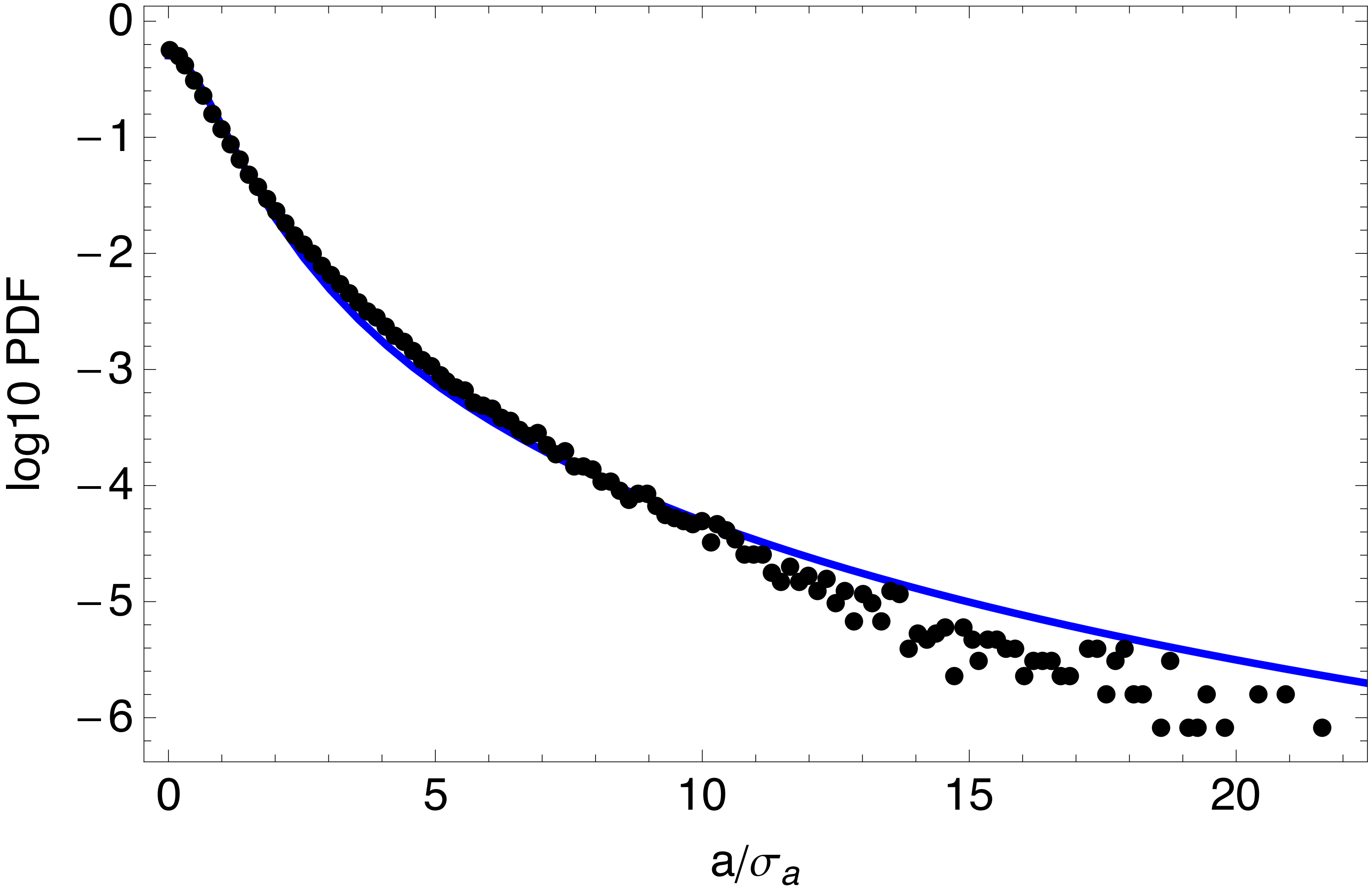


Classical (blue) vs. quantum harmonic oscillator (red): velocity PDF



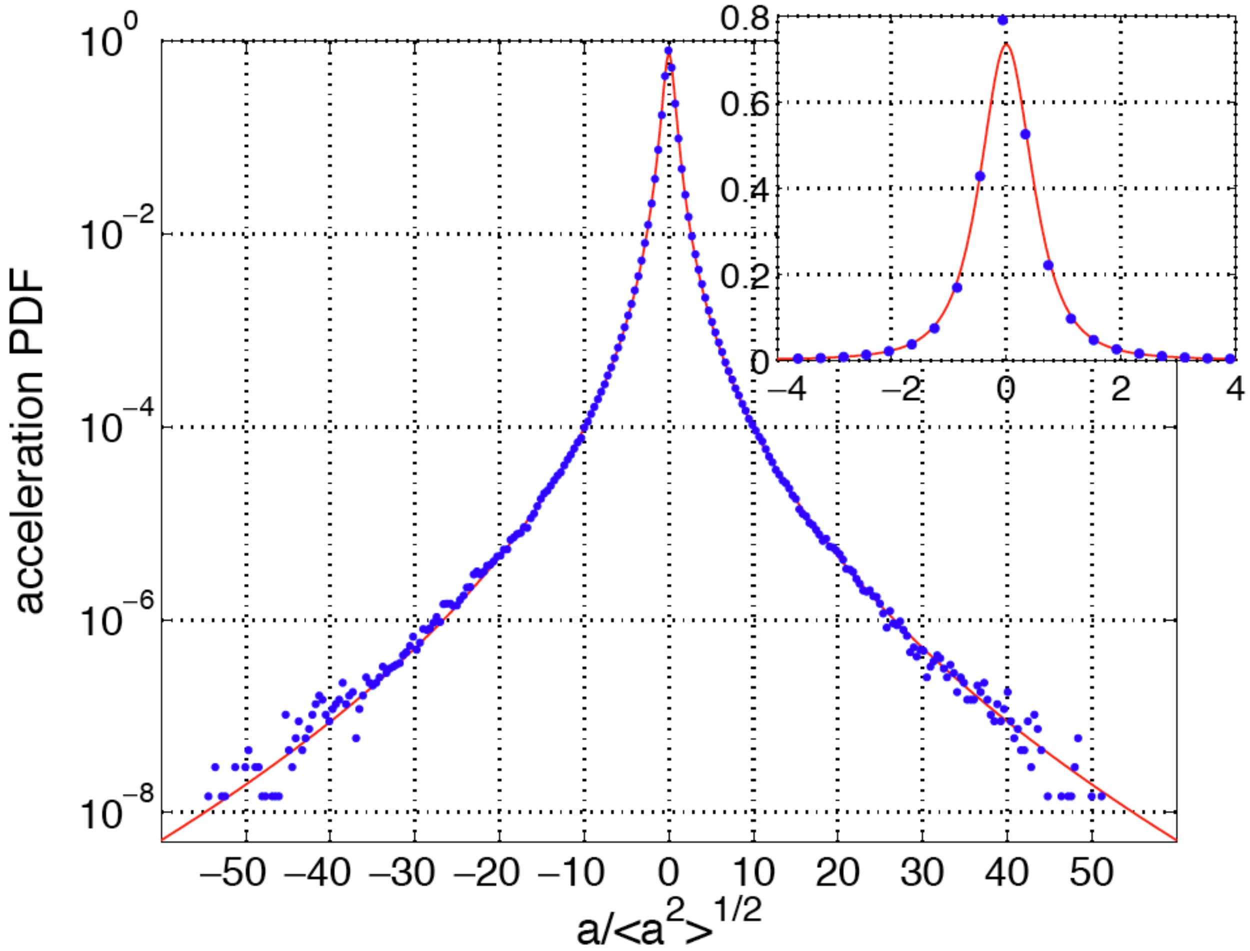
n=3; v_0 = 0.3 m/s

Comparison P(a) with experimental data L (but here surestimate at large a)



N. Mordant 2001 PhD thesis. Mordant et al PRL 2001.
Double von Karman flow; Lagrangian tracers 250 μm ; ultrasonic doppler tracking; sampling 6.5 kHz == 0.7 τ_η (0.22ms); R_λ = 810; fully developed turbulent flow.

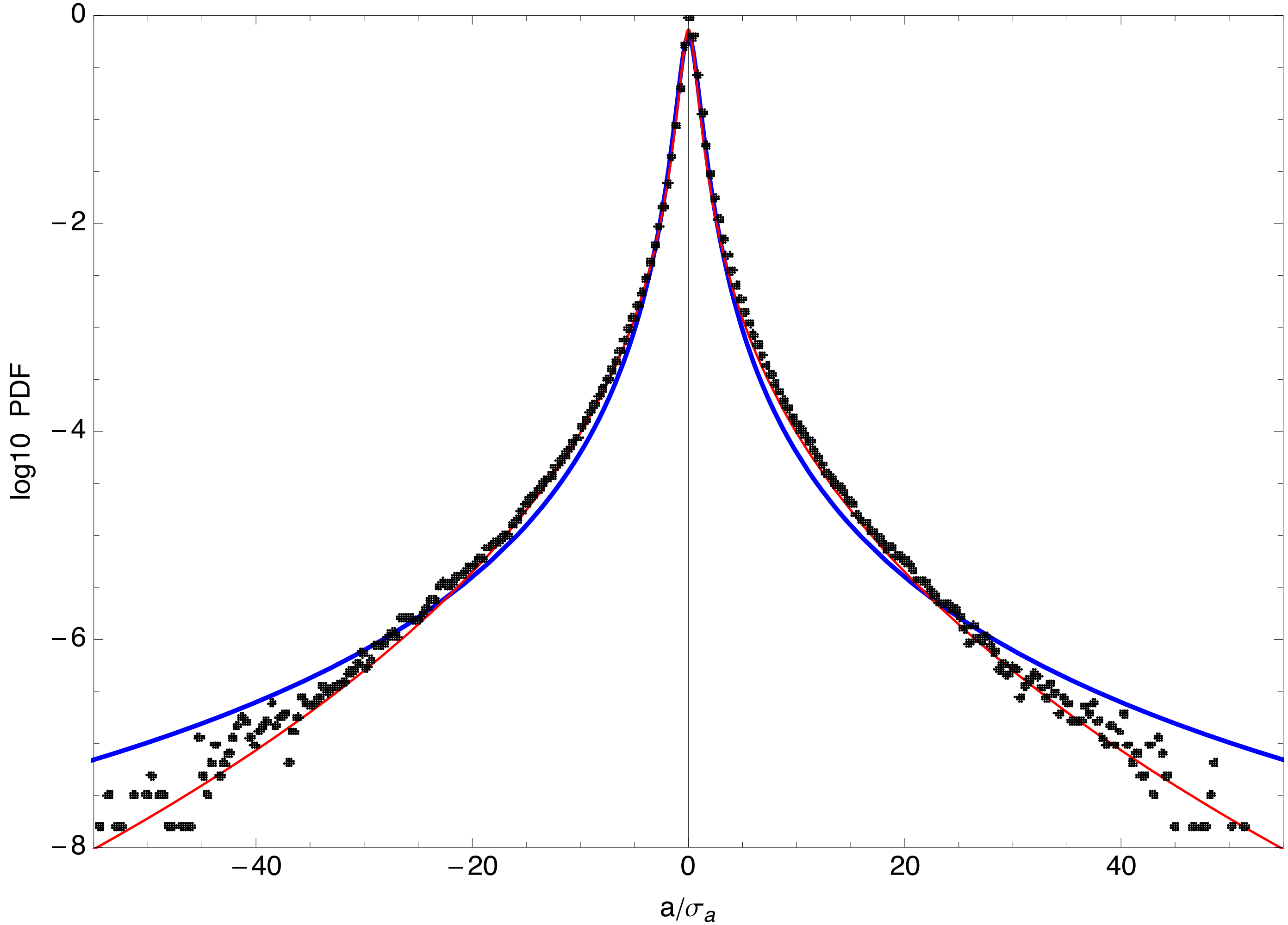
Lagrangian PDF(a): experimental large tails



Double von Karman flow; two counter-rotating disks, d=20 cm, 33 cm apart; Lagrangian tracers 46 μm ; silicon strip detectors; sampling 70 kHz == τ_η (0.93 ms)/65; 170 million pts; R_λ = 690; fully developed turbulent flow.

Ref.: Mordant -Crawford -Bodenschatz ; Physica D 193 (2004) 245–251

Experimental data vs. Fit stretched exponential vs. $1/a^4$ law

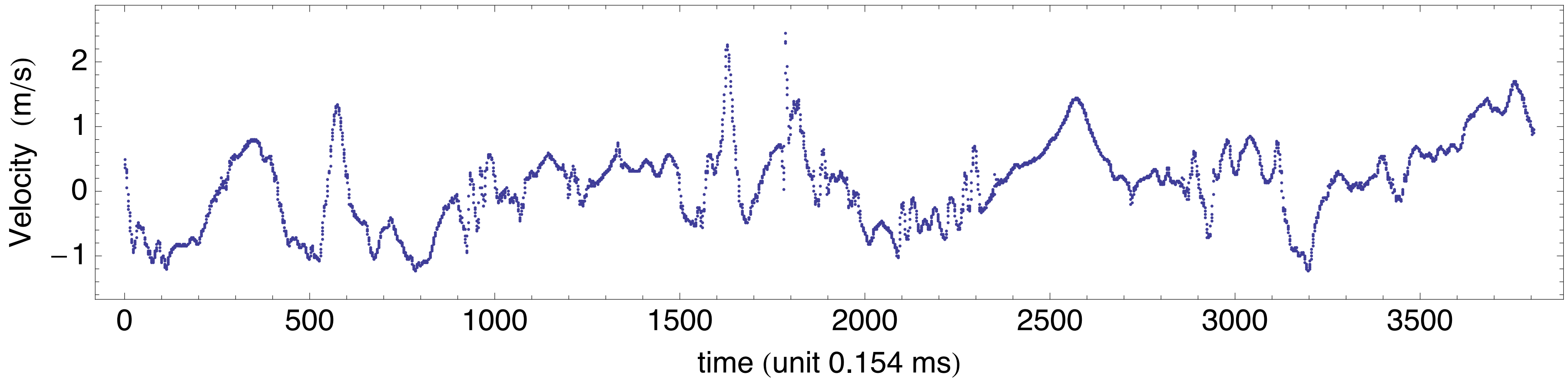


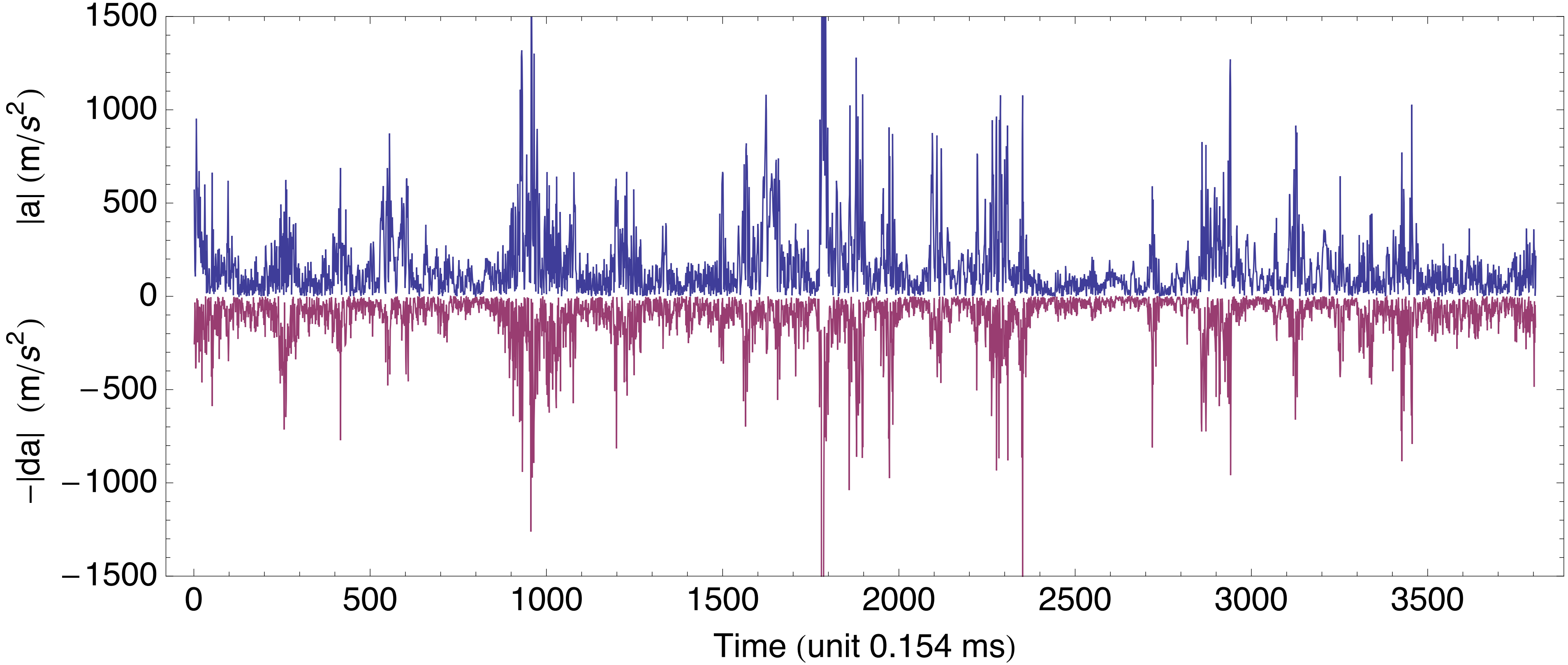
*Red: stretched exponential (Voth et al 2002, Mordant et al 2004)

*Blue: PDF (A_q) *alone* --> to be added:

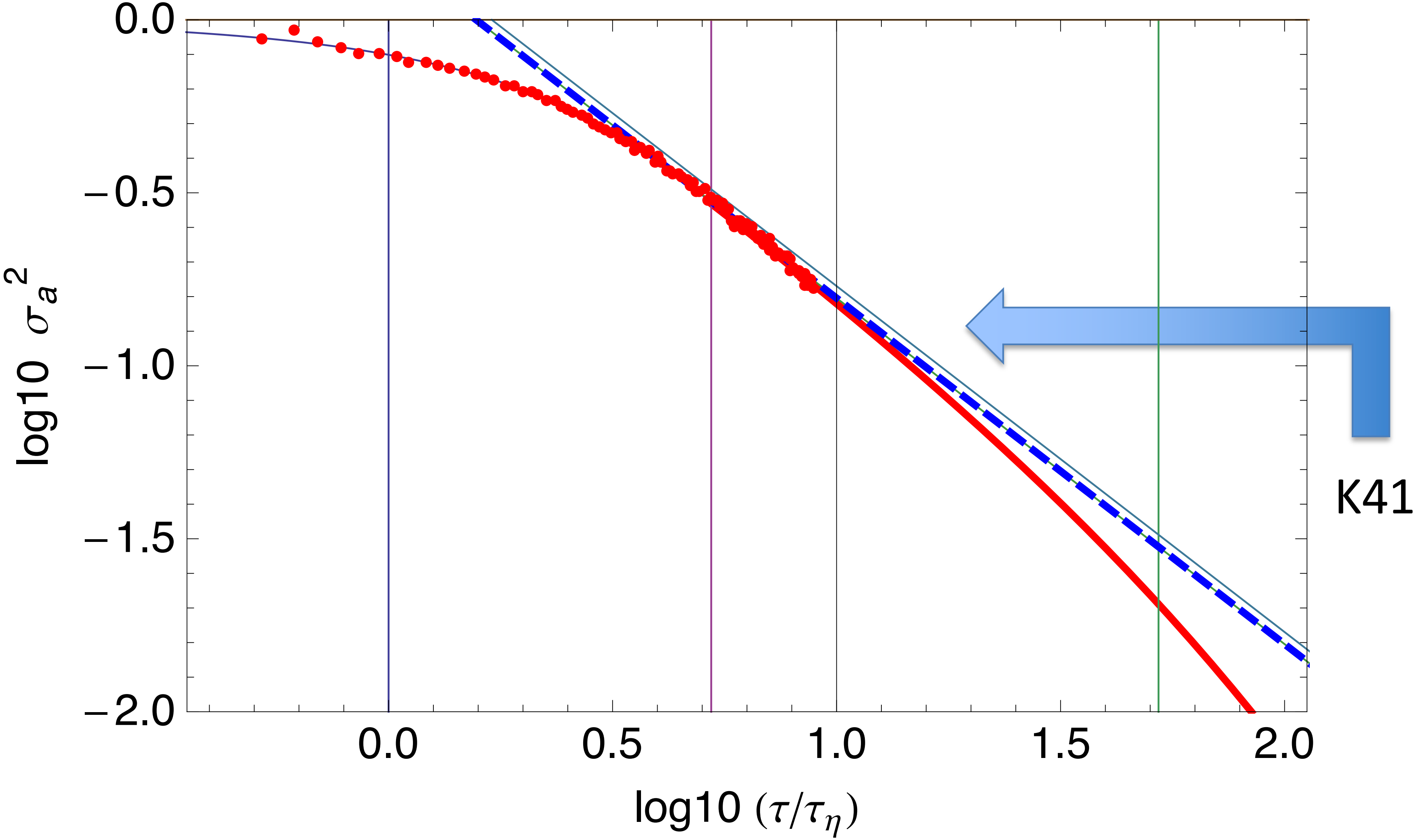
- effects of dissipative Kolmogorov transition
- other contributions

Fine analysis of « intermittence » : on Mordant data L, seg3398



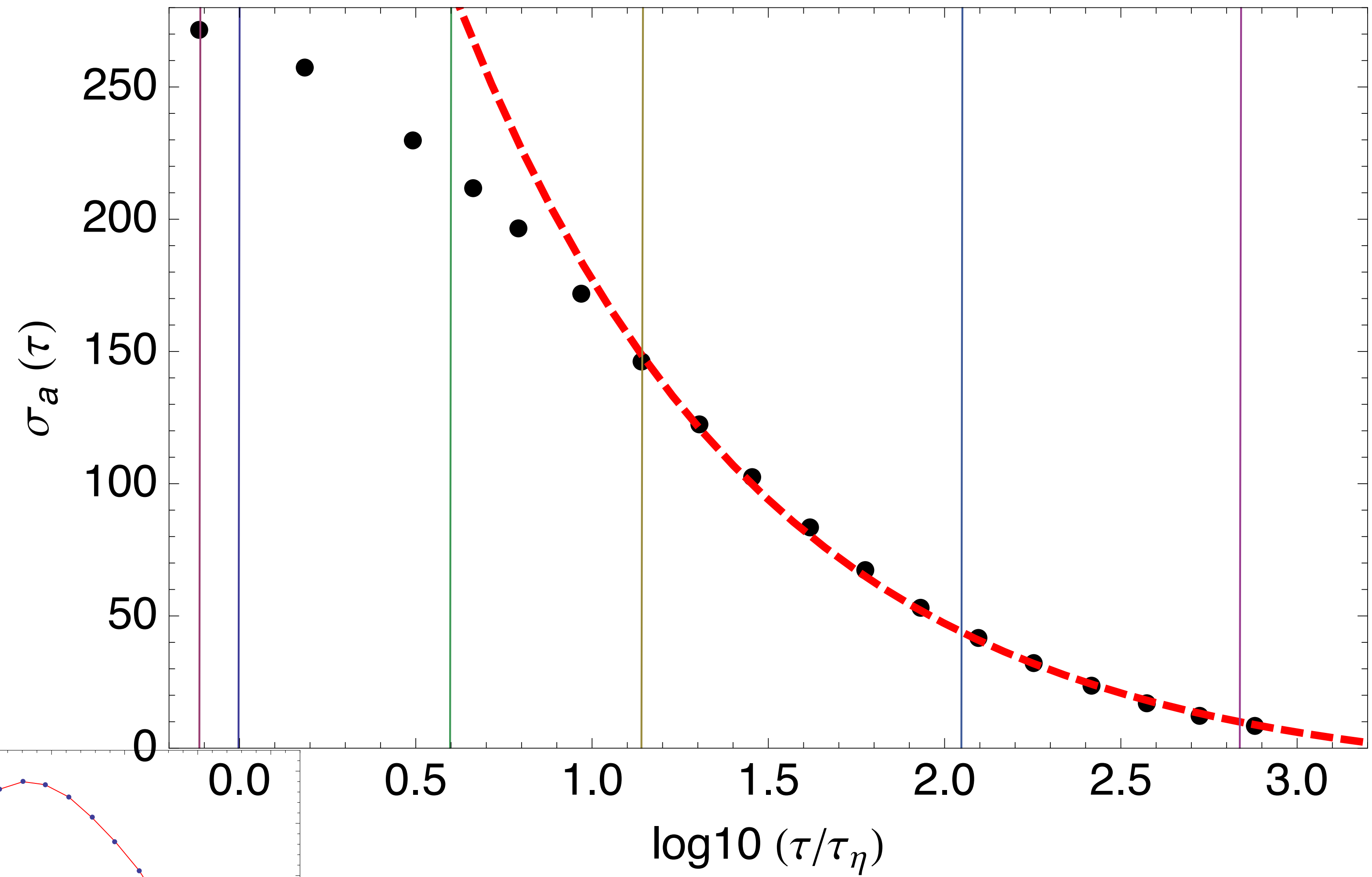


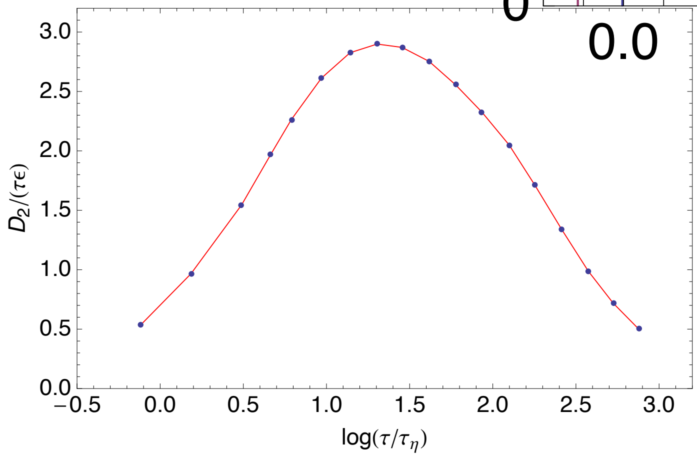
|a| ≈ |da| : justification of two-valuedness of acceleration



Voth et al 2002; Rλ = 970.

Mordant, man290501.



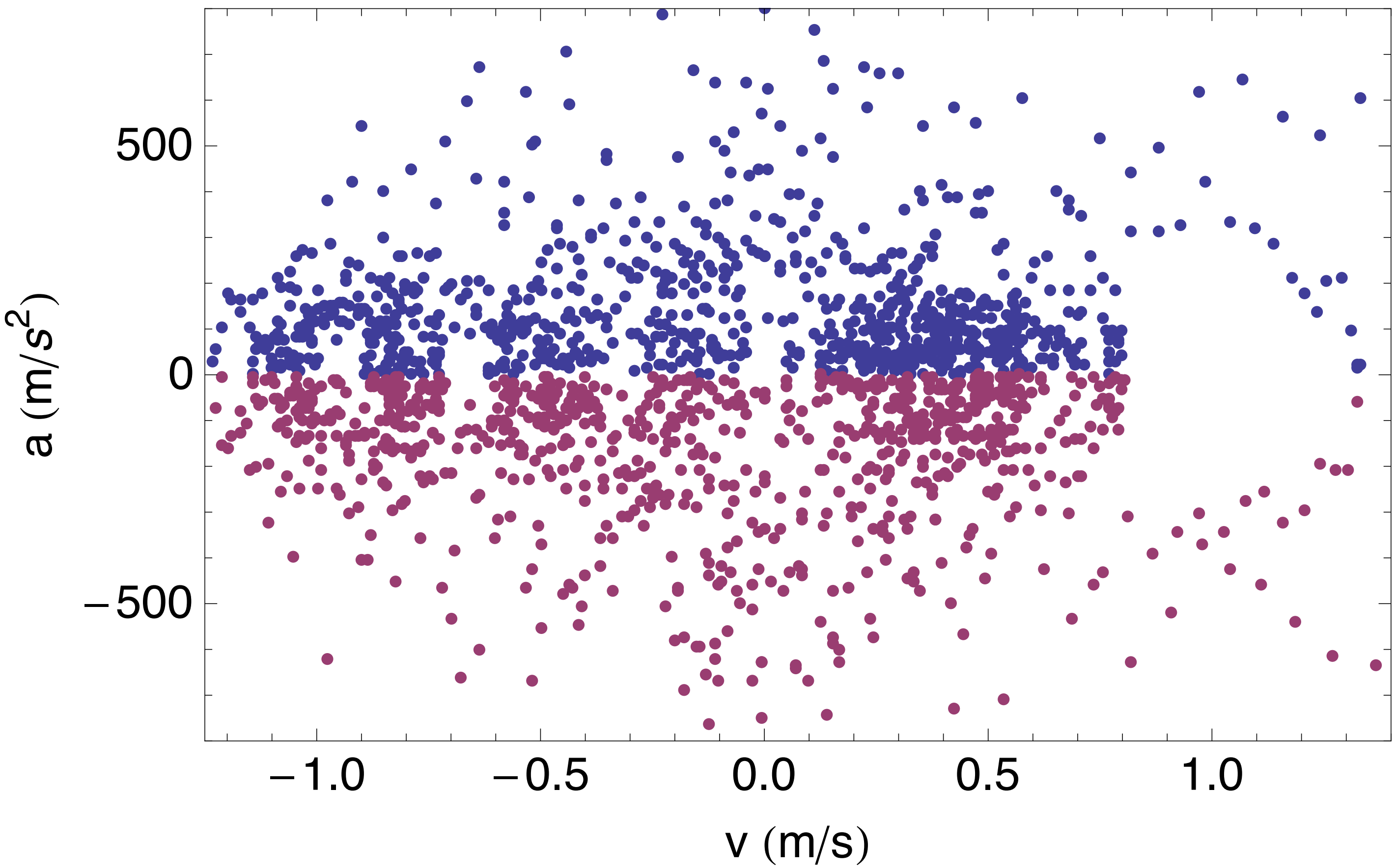


Points: data

Red dashed line: K41

v-phase diagram (*v*,*a*)

Mordant data: exp man290501; Seg3398 : t = 1-1770 t_u , t_u=0.7 t_K

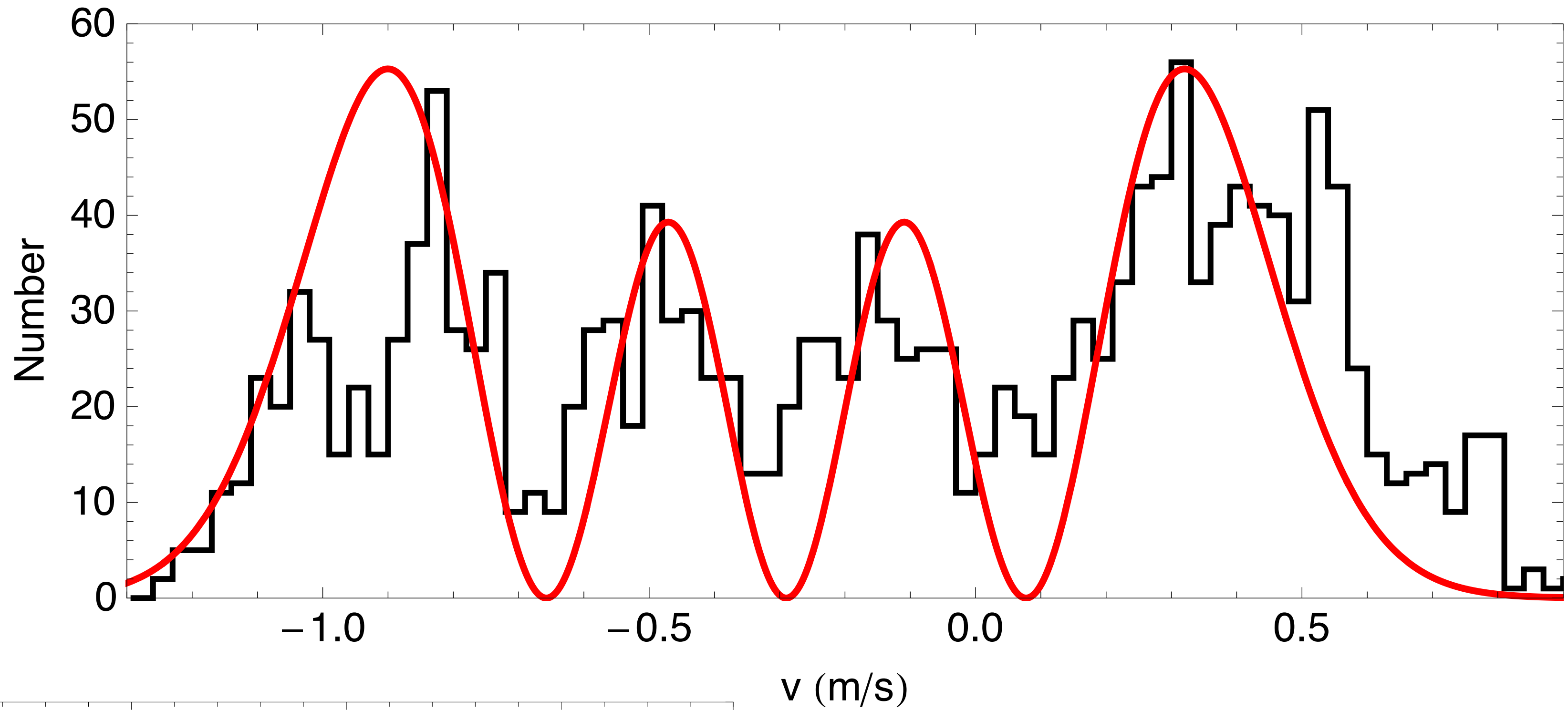


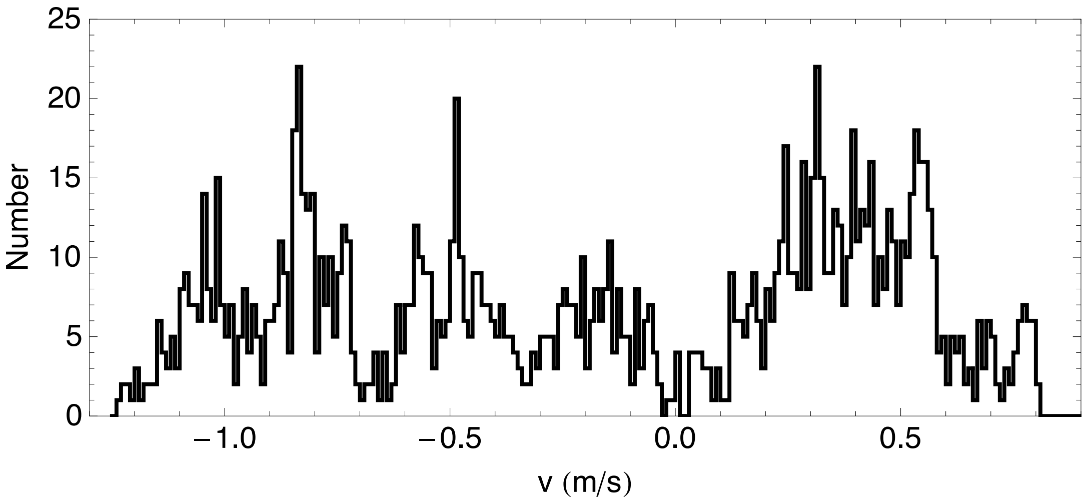
Validation of existence of values of *v* for which P[*v*]=0, especially for |a| < σ_a = 240 m/s^2
(v=0 : partially due to bias in 1D data, better on 2D data) , quantum-classical transition

Local quantized harmonic v-oscillator Φ(v)

Seg3398-1-1770: n=3, v_0=0.3

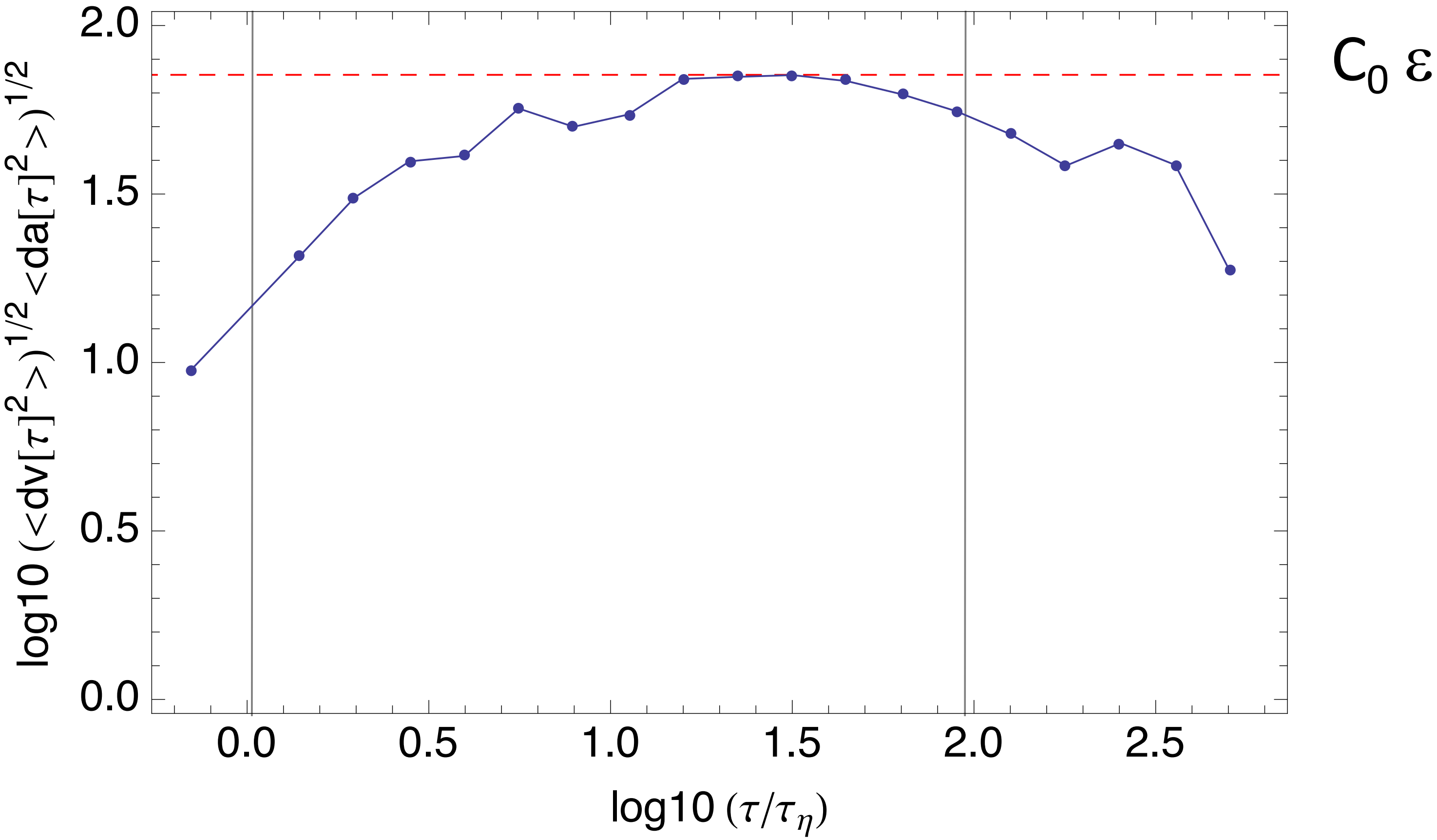
All seg ->





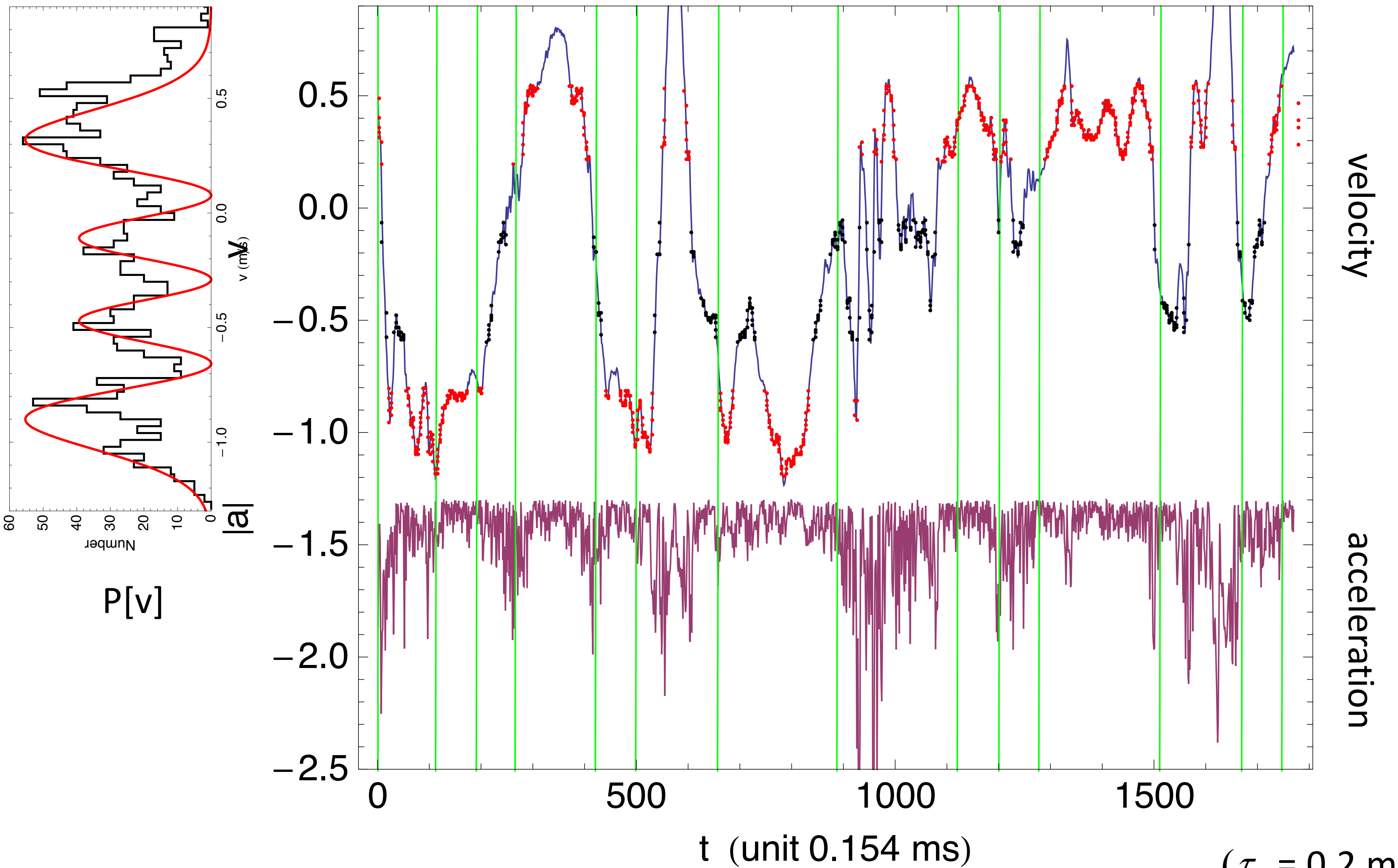
← Account of Transition cl-qu $|a| < \sigma_a$

Heisenberg-like relation in *v*-space : $\delta v\, \delta a \approx \hbar_v \approx$ cste



K41: $\hbar_v = C_0\, \varepsilon = 2\, \sigma_v^2\, /T_L$

Mechanics of acceleration intermittence

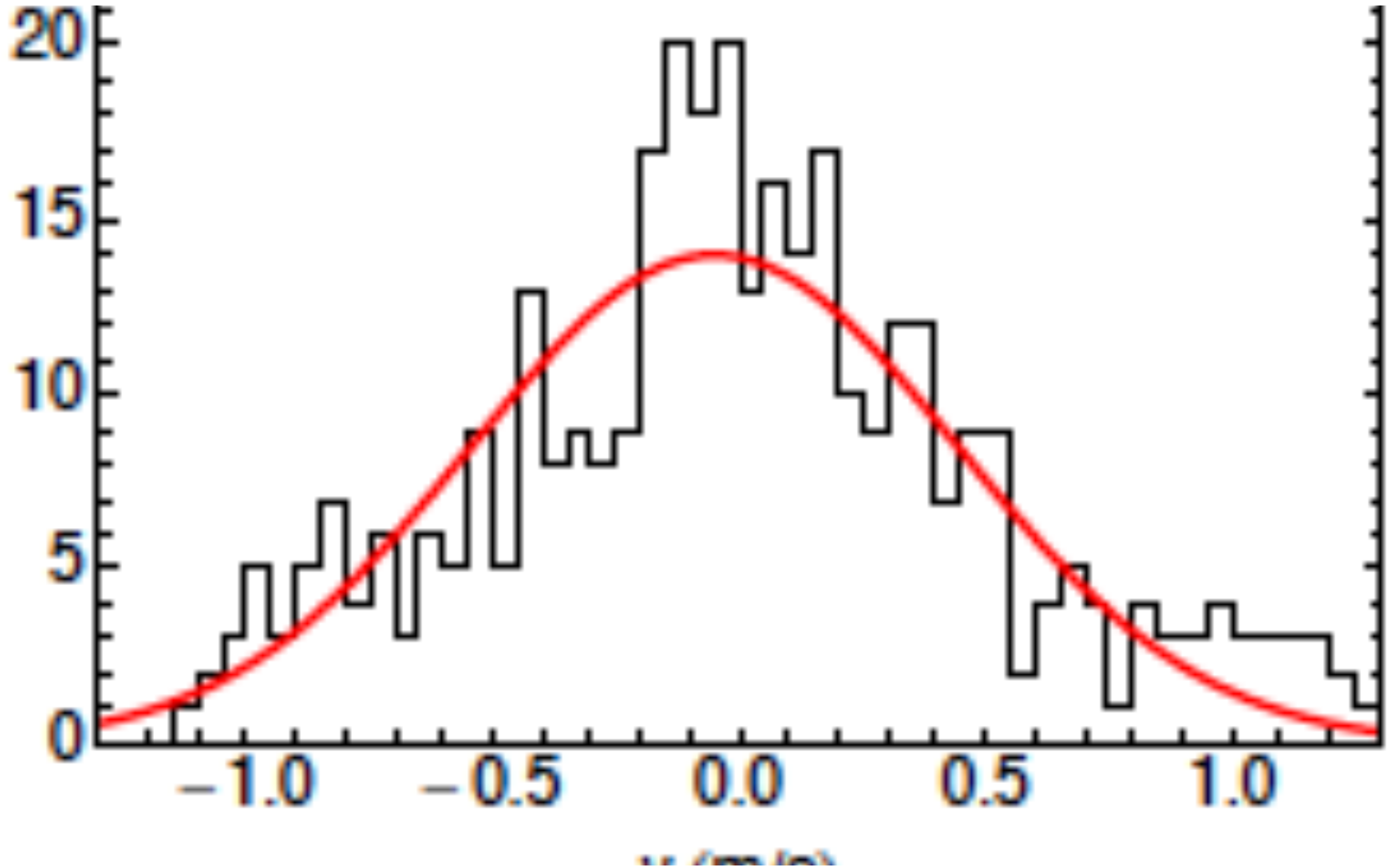


(τ_η = 0.2 ms)

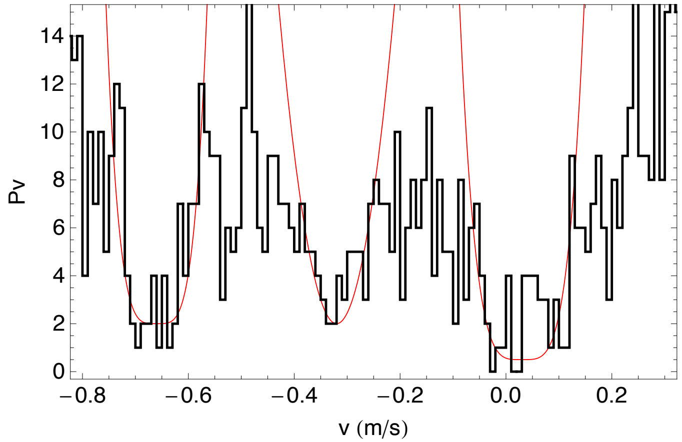
(TL = 22.4 ms = 146 tu)

Quiet periods = particle trapped in main probability peaks
Bursts = macroquantum jumps between probability peaks

Mechanics of acceleration intermittence, Gaussian shape in classical zone for P(v)

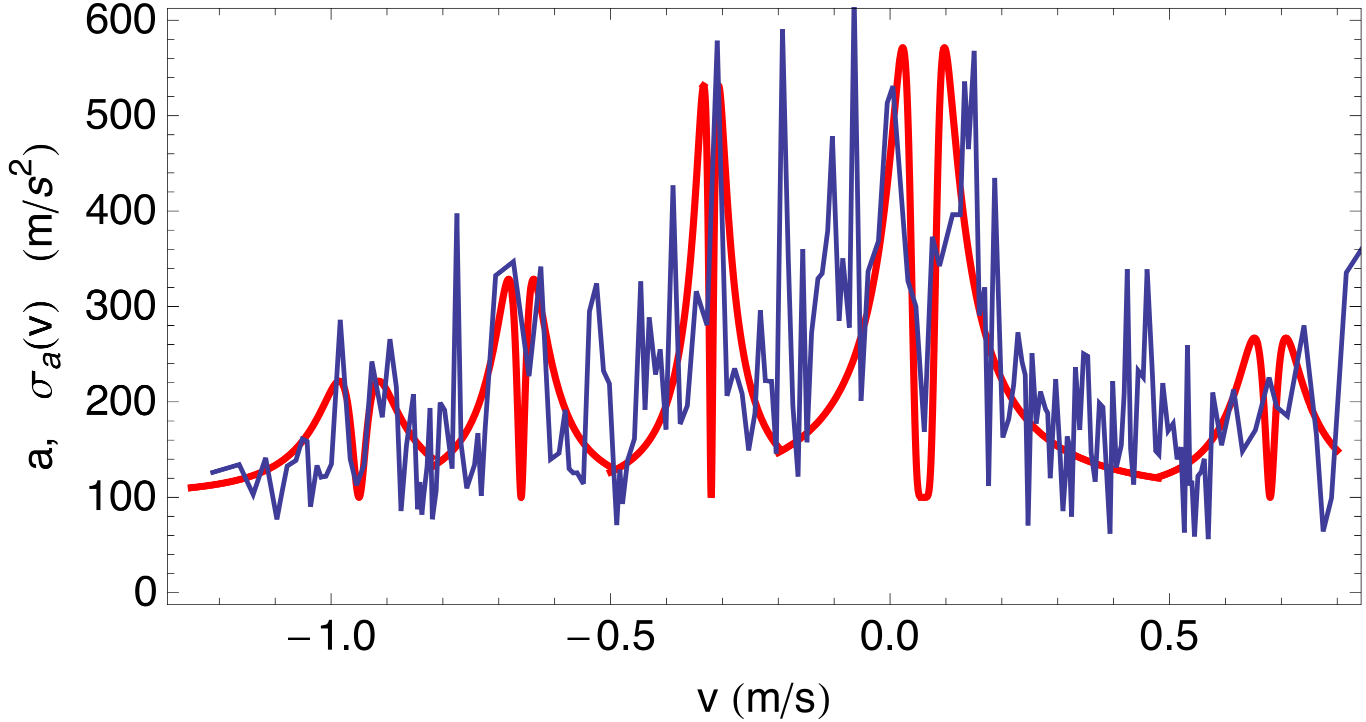


Modelization of minima of P(v) –> $A_q(v)$



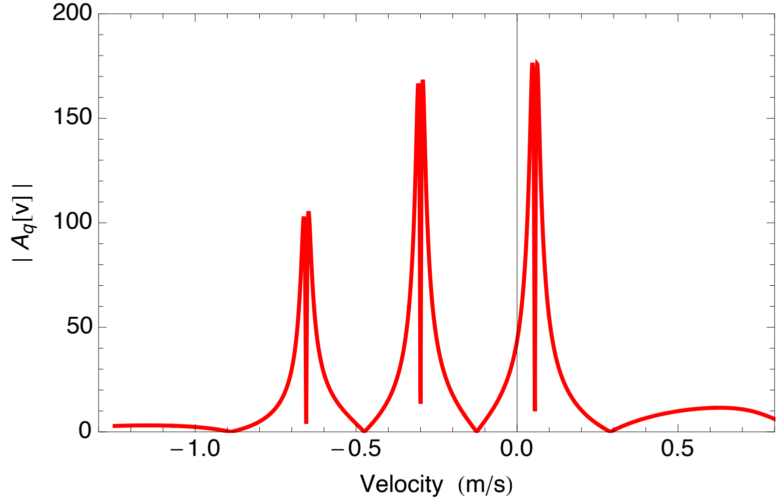
Blue: observed running acceleration dispersion in function of velocity (partition : 9 points)

Red: predicted value of $|A_q|(v)$ (by numerical simulation)

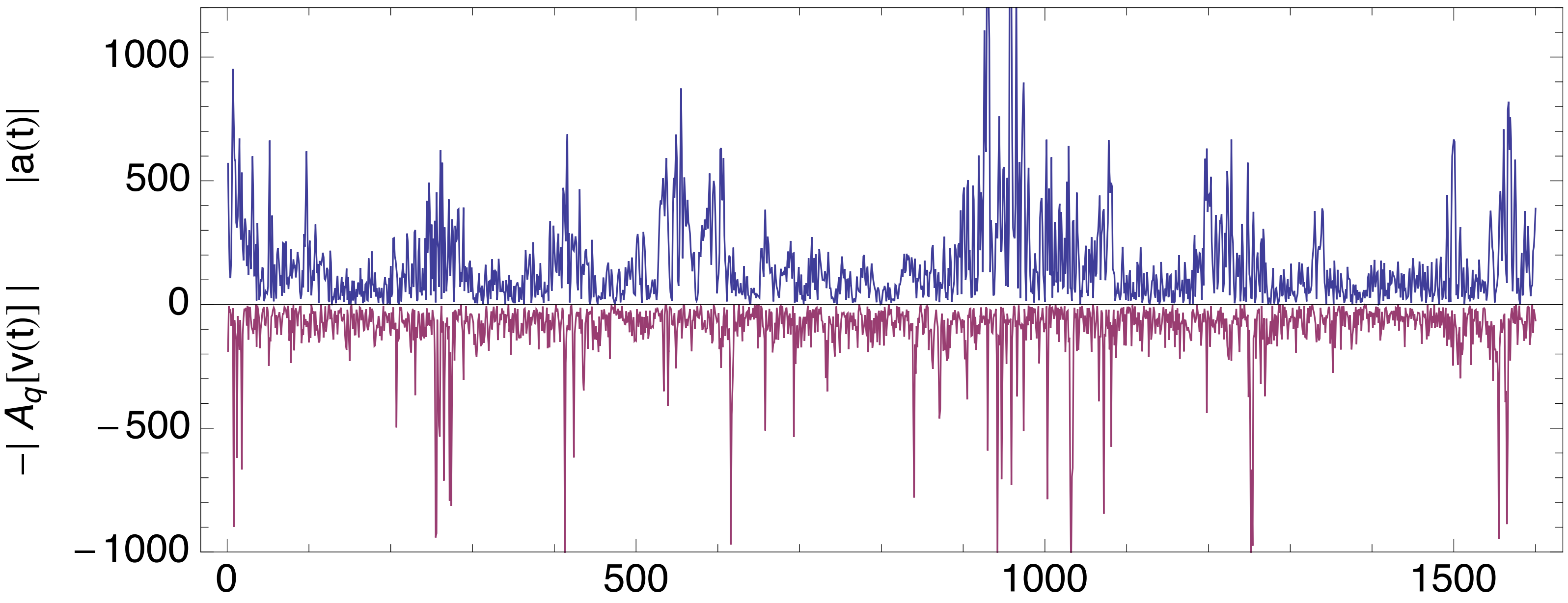


Fit of P[v] by quantized harmonic oscillator
–> now $A_q[v(t)]$ vs a(t)

Theoretical model: Aq (v) from
(quantized harmonic oscillator n=3)
times Gaussian (global velocity PDF)



Predicted |Aq[v(t)]| compared to observed |a(t)|



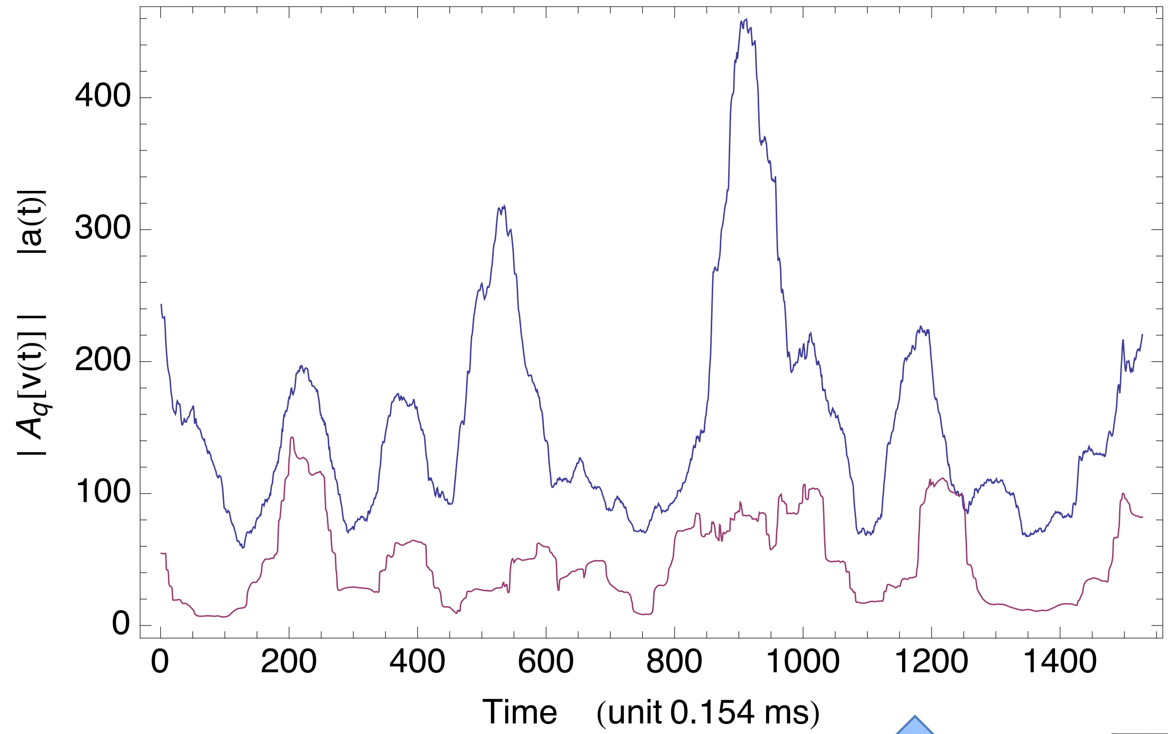
← observed

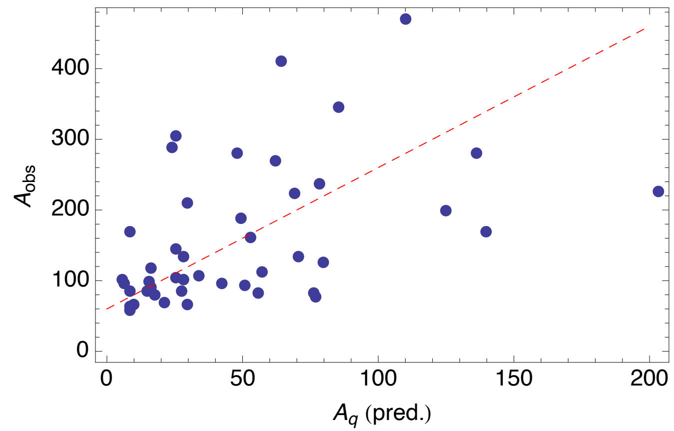
← predicted

Added: $\sigma_{a\xi}$ =80 m/s^2

Time (unit 0.154 ms)

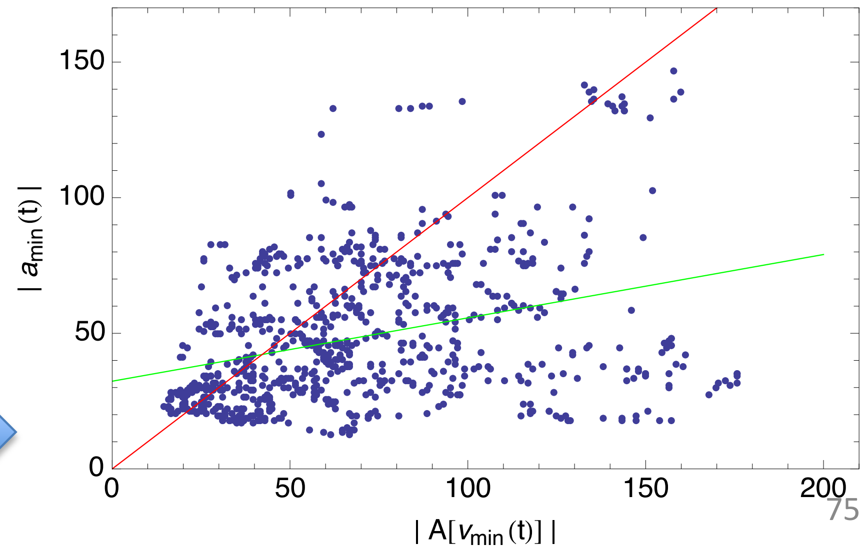
Predicted $A_q[v(t)]$ vs observed a(t) : best with time averaging





Averaging on $\Delta t = T_L/2 = 73$ tu

Aq[v(t) vs minimas of bursts of a(t).
Statistical significance of correlation :
10.1 sigmas



Consequence for stochastic equations SDE (1)(TBC)

- Second / third order SDEs : completed eqs. (but still not fully consistent....)

New component Aq: pseudorandom

$$\frac{dv}{dt} = -\frac{v}{T_L} \pm \mathcal{D}_v \nabla_v \ln P_v + \sqrt{C_0\,\varepsilon}\,\frac{dW_v}{dt},$$

$$\frac{da}{dt} = -C\,a - D\,v + 2\mathcal{D}_v^2\,\nabla_v\left(\frac{\Delta_v\sqrt{P_v}}{\sqrt{P_v}}\right) + B\,\frac{dW_a}{dt} \;\; - \omega^2\,(v - v_0)$$

New: macroquantum potential
--> Schrödinger equation

Dissipative oscillator (TBC...):
exact solutions of viscous NS eq.

b=da/dt → db/dt = - ω^2 *a* + dW_b/dt (3rd order SDE)

Ref. : LN, 2014, in *Space-Time Geometry and Quantum Events*, I. Licata Ed., Chap. 5. , p.175-196 (Nova Pub., New York).

SDE (2)

- Here we start with SDE(0) recall below and a Gaussian(G) noise and have found a new but predicted Aq acceleration component (see SDE (1))
- $$dv / dt = -v/T_L + dW_v / dt$$
  $$da / dt = -a/T_a - a/T_a T_L + dW_a / dt$$
- But this is « orthogonal » with « usual » approach of turbulence with SDE where all the Non Gaussian feature is set in phenomenological functions put in front of the dW term…

Partial conclusions and prospects

(Validation of de Montera's proposal with high statistical significance)

- **P(v) = local quantized harmonic oscillator solution of Schödinger-type equation (and also classical forced /damped oscillator) --> the new suggested turbulence mechanics is a mixing of classical and « macro-quantum » mechanics**

$A_q(v)= D_v\, \partial_v \ln P_v$, new acceleration component with major contribution to large tails of strongly non-Gaussian acceleration PDF: validated to 10 sigmas significance.

$$\sigma a^2 = \frac{\sigma v^2}{TL^2} + \sigma Aq^2 + \sigma A\upsilon^2 + \sigma a\xi^2$$

Prospects :

- Local study on other segments, other experiments (different Re numbers)

- Global study : looking for macroquantum global structures

-Account of : *Scale dependence : $D_v = D_v(\tau)$; *(Macro)-quantum / classical transitions;

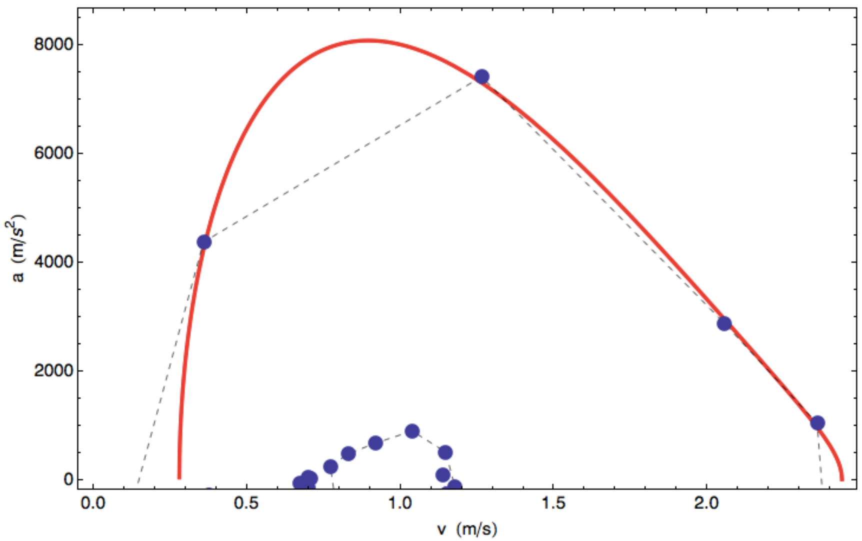
*Dissipative Kolmogorov transition around τ_η : dissipative eddies

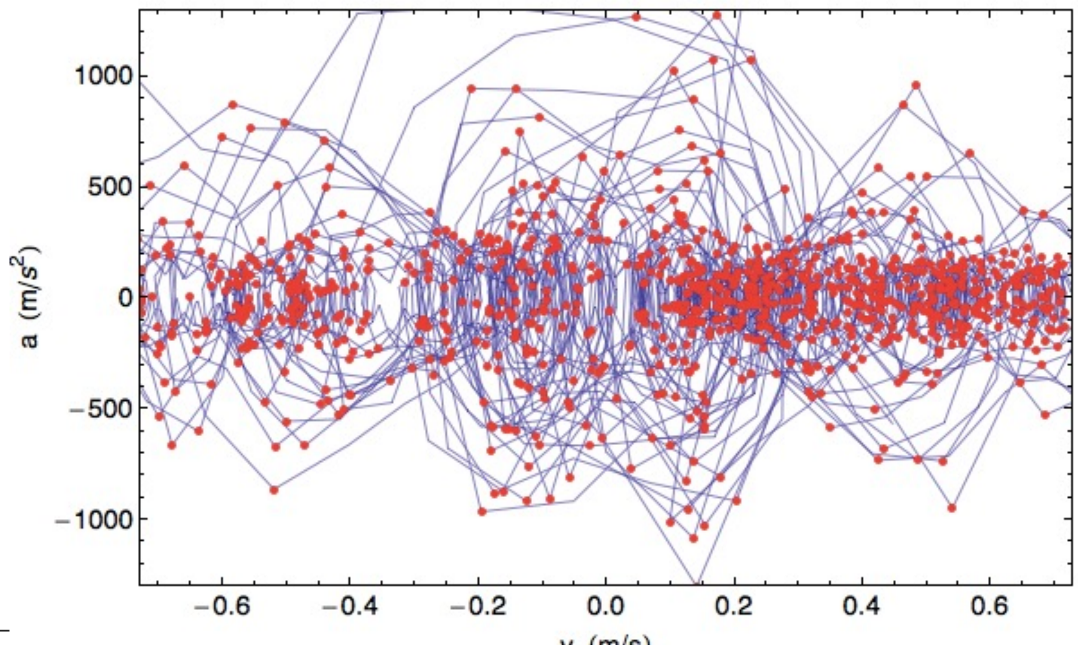
*...other effects, DNS , Other Data…

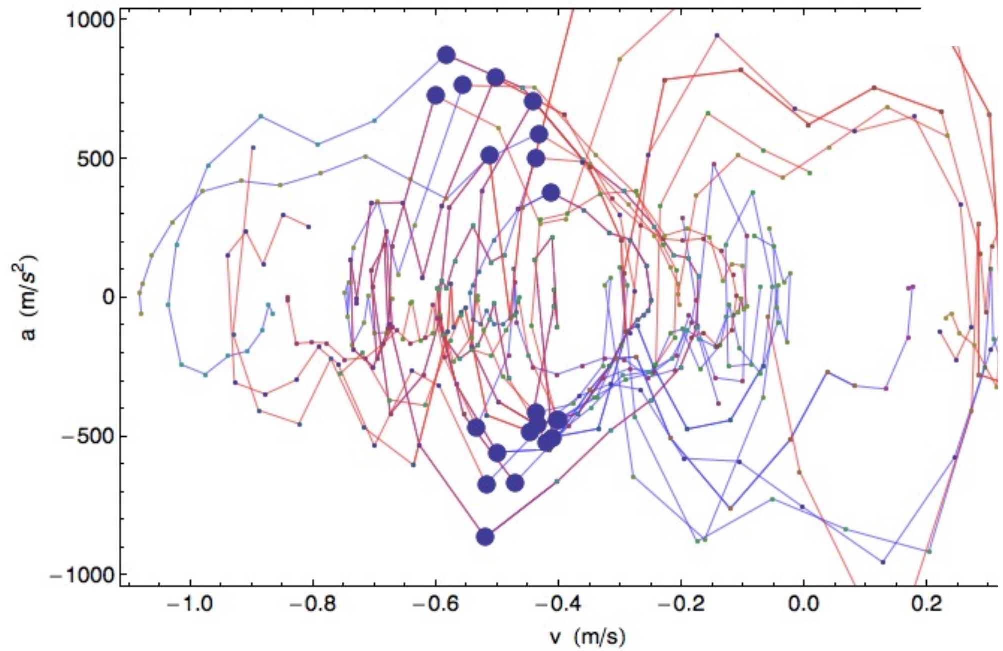
Role of arches ((an)harmonic oscillators in the dissipative domain)

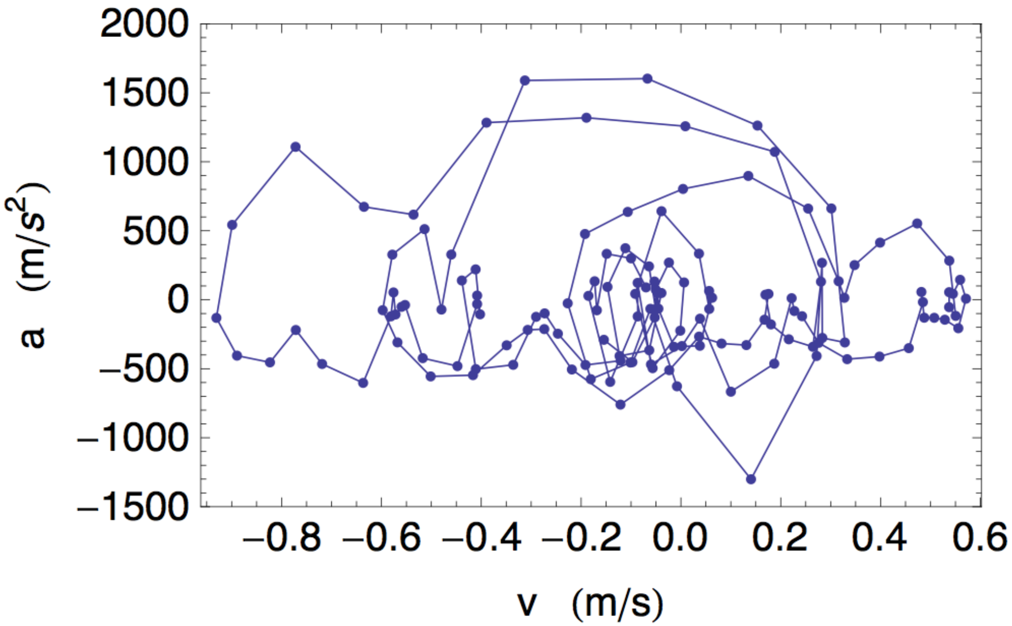
- Not explained in detail here but the arches allow to correct the pdf(a) for small a and for large a (with an exponential cut-off)
- See an exemple on next slide +applications

« Arches »: examples in experimental data

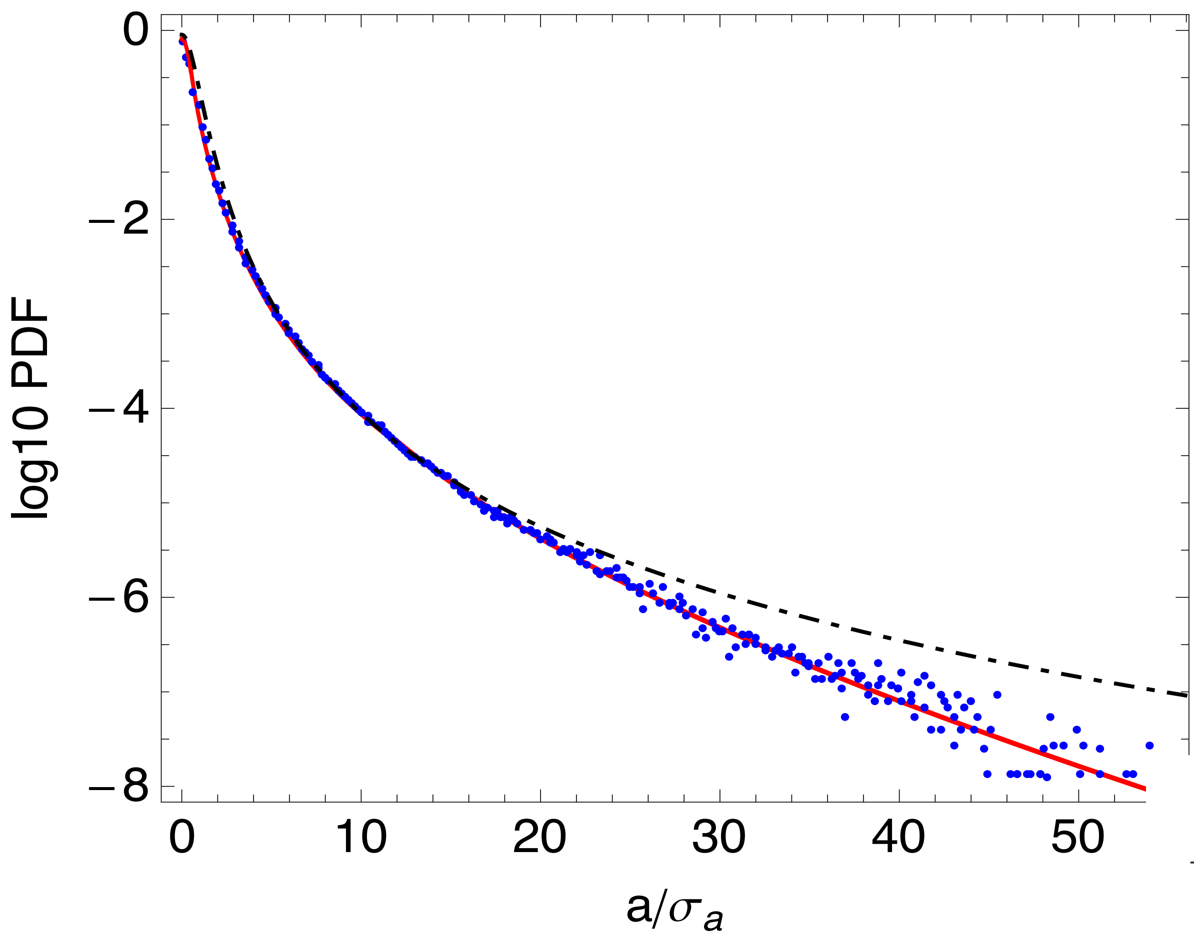








Acceleration PDF : comparison to a^{-4} theoretical prediction



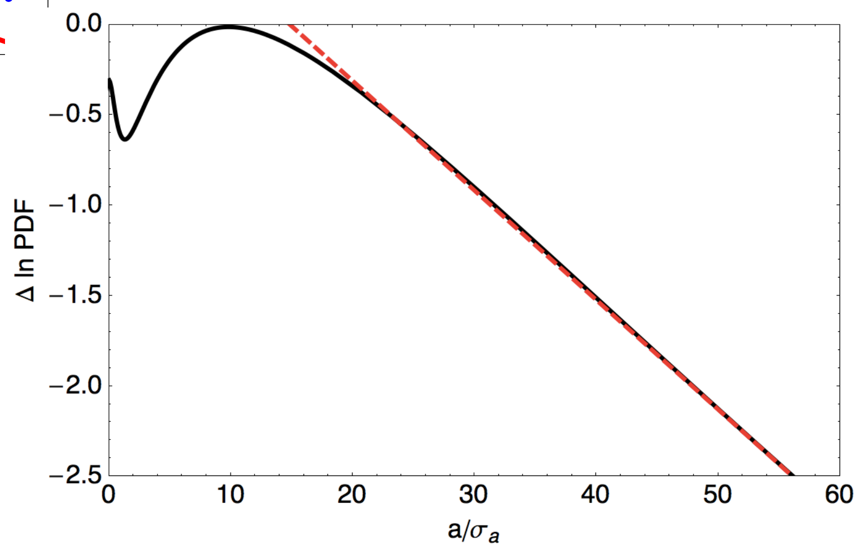
<——

Blue :
Bodenschatz data

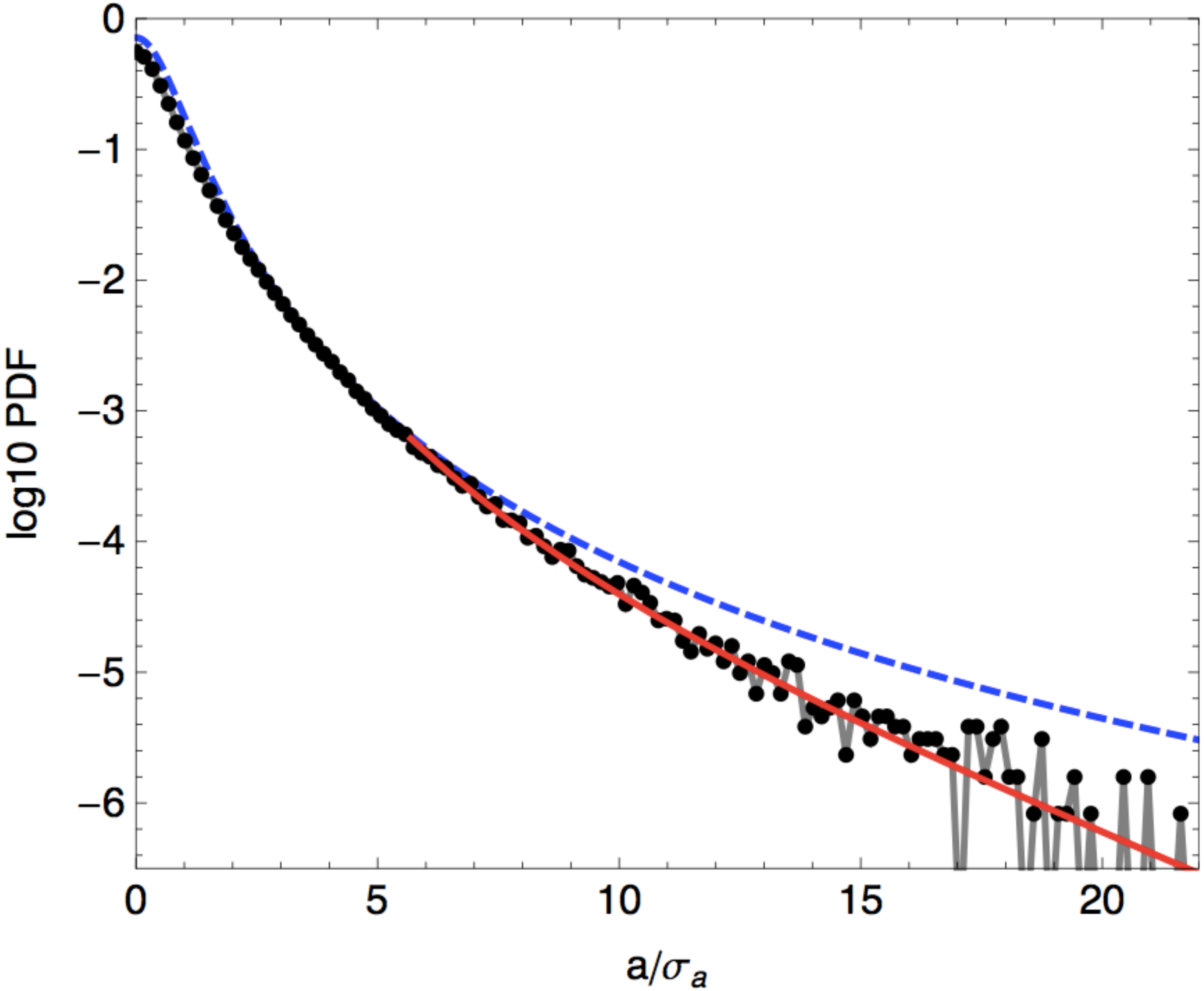
Red :Phenomenological fit
(stretched exponential)

Black dashed :
$(1+a^2)^{-2}$ law

Difference witt a^{-4} law →
Black: fit of data
Red dashed: exp cut-off

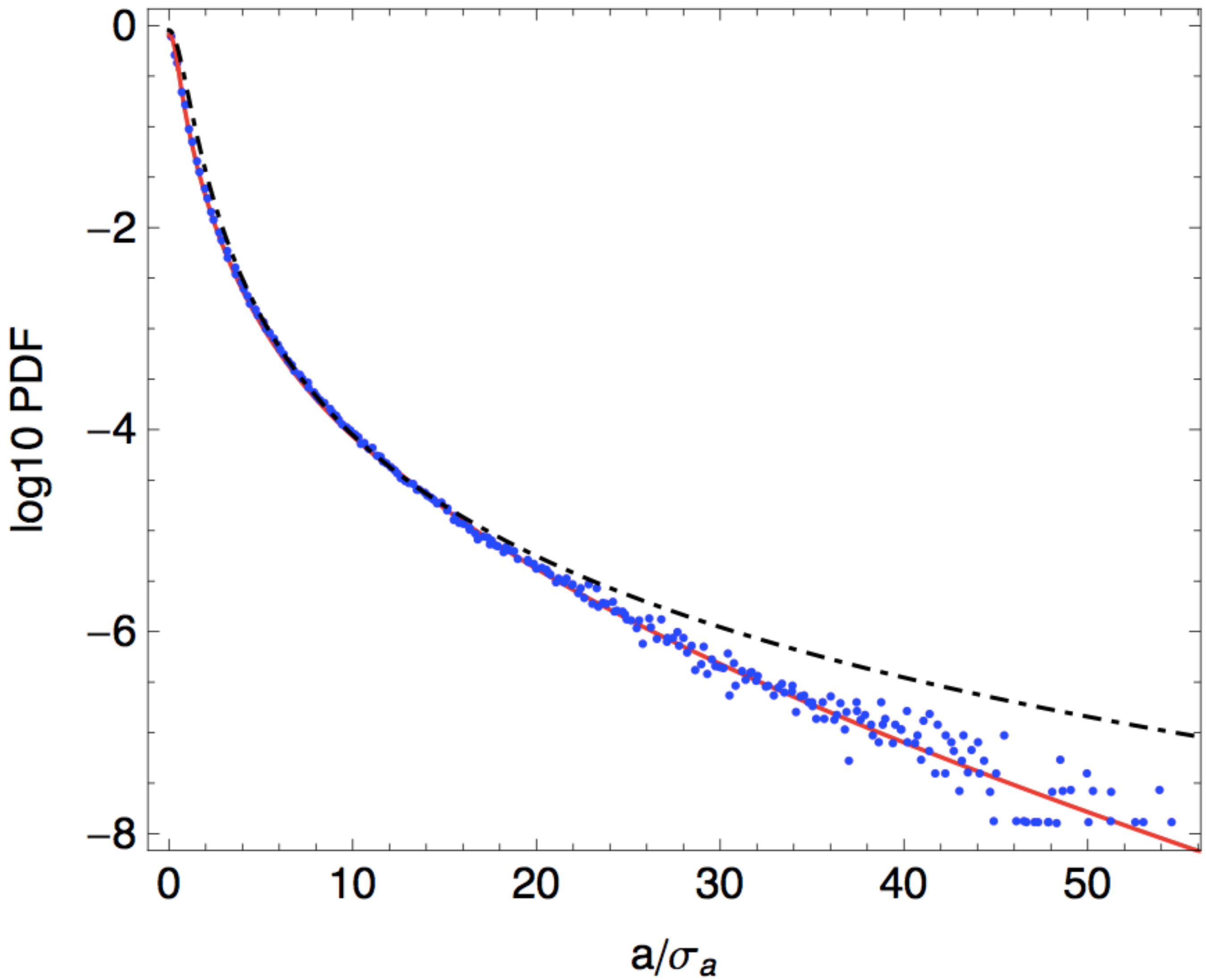


Comparison with experimental data with arches



Mordant data : excellent fit by theoretical prediction

Comparison with experimental data: combined « small » and large acceleration corrections



Bodenschatz et al. data : excellent account of large tails up to > 50 sigs!

CONCLUSION: doubly corrected 1/a^4 law => perfect fit of the whole PDF of acceleration

Messages for part IV

5) new recent applications of scale relativity

- - for rotating turbulence in geophysics : Schrodinger eq. in a vectorial field (good agreement for pdf(a) with oceanic data)

- -for turbulent free jet and shear flows
- ADD: (main features explained :angle, closure pb, ...)

- -for turbulent boundary layers
- ADD: (prediction of Karman cte...)

- + our own experiments on free jets...

6) Conclusions

- SR useful to foundation of QM with underlying stochastic motion
- Success to find mecanisms and description of HIT turbulence /+new applications of section 5)
- Looking further for other applications in compressible turbulence (density could be also fractal itself) in geophysics (for fluids in rotation) and in astrophysics like in MHD compr ou incompr (with density, velocity and magnetic fields as possible stochastic/fractal variables)
- Experimental work in progress with free jet

5) More if time on ongoing work:

New prediction : a) application of SR to turbulence in rotation

The NS incompressible equation of motion in the rotating frame reads :

$$D_t u = -\nabla(p + p_c) / \rho - 2\Omega \times u + (\eta \Delta u)$$

$$D_t \rho = -\rho(\nabla.u)(CP = 0)$$

$$D_t = \partial_t + u.\nabla, p_c = \rho(\Omega \times r)^2 / 2$$

If again u is fractal in the sense du^2 in dt we can write now a Schrodinger equation in the u space but in presence of a vector potential Ac(u) to account for the Coriolis force such that Curl(Ac)=2Ω x u

5) SR in rotation (2)

- We can now predict like before an acceleration A=-2i$D_u$grad$_u$(Ln(Ψ(u(t,dt),t))   (I)
- But now Ψ is solution of a (vectorial) Schrodinger eq. with Ac(u) :

(3)

$$(iD_u\partial_t + (iD_u\nabla_u - A_c(u,t))^2 - \Phi(u,t)/2)\Psi(u,t) = 0$$

Solutions can be found within 3 harmonic oscillators : Φ external potential, Φc centrifugal potential (also harmonic) and 3nd potential which comes here from the Ac potential expression.

Solving for stationnary states with HΨ=EΨ, H Hamiltonian is here symmetric in vx,vy variables

For a 2D case we choose in cartesian coordinates Ac=(-Ωy, Ωx,0) for Ω//ez

- exact determination of the eigenfunctions and eigenvalues E(Lz) involving the kinetic momentum Lz=v_xd/dv_y-v_y/d/dv_x for eq.(3):

5) SR for Rotation (3)

$En=(n+1/2)E_0+f(lz,lz^2)$

Ψn,nϕ,pz=Ψn(r)exp(ilzϕ) exp(ipz.z)

Ψn(r)=cn Hn(r).exp($-r^2/2$) with here r=vr, ϕ=arctan(vy/vx), z=vz

A+,-=(A_r, A_ϕ, A_z)= (+-Dv dr(P(vr))/P(vr), 2D lz/vr, 2D pz)

P(vr) like at 1D but in radial direction

Here H is symmetric thus the accelerations are independent of the polar angle ϕ.

But now the minima of Pv(vr) are on a circle in the plane

(vr, vϕ) instead of a single point in 1D case

Thus we expect here again a pdf(a) in $1/a^4$ for a=ar…

5) SR in rotation (4)

Need of data from experiments and/or from DNS to test predictions on P(v) and P(a)…

- More involved cases :
- if Ω is not uniform : Ω(x)->Ω (u)? to account for ex of a velocity shear
- ….

Other work in progress : HD compressible case (1)

- NS or Euler compressible case (even for Ω=0) : the density may be itself a fractal function through x(t,dt) and/or by itself…

$$D_t u = -\nabla(p)/\rho + (\eta \Delta u)$$

$$D_t \rho = -\rho(\nabla . u)$$

$$D_t = \partial_t + u.\nabla$$

- Brownian in x : use of the covariant derivative applied to u but also to ρ :D_t->$D\hat{}_t$

HD compressible (2)

$$\hat{D}_t u = -\nabla(p)/\rho + (\eta\Delta u)$$

$$\hat{D}_t \rho = -\rho(\nabla.u)$$

$$\hat{D}_t = \partial_t + U.\nabla - iD_{\mathrm{x}}\Delta_{\mathrm{xx}}$$

$$U = -2iD(\nabla Ln(\Psi(\mathrm{x},t)))$$

$$\hat{\rho} = -2iD(\nabla Ln(\Psi_{\rho}(\mathrm{x},t)))? \text{ or } \hat{\rho} = \left|\Psi(\mathrm{x},t)\right|?$$

- Next develop this case … in x space then in u space +fractal intrinsic possible also for ρ(x,t)…

MHD case (in course)

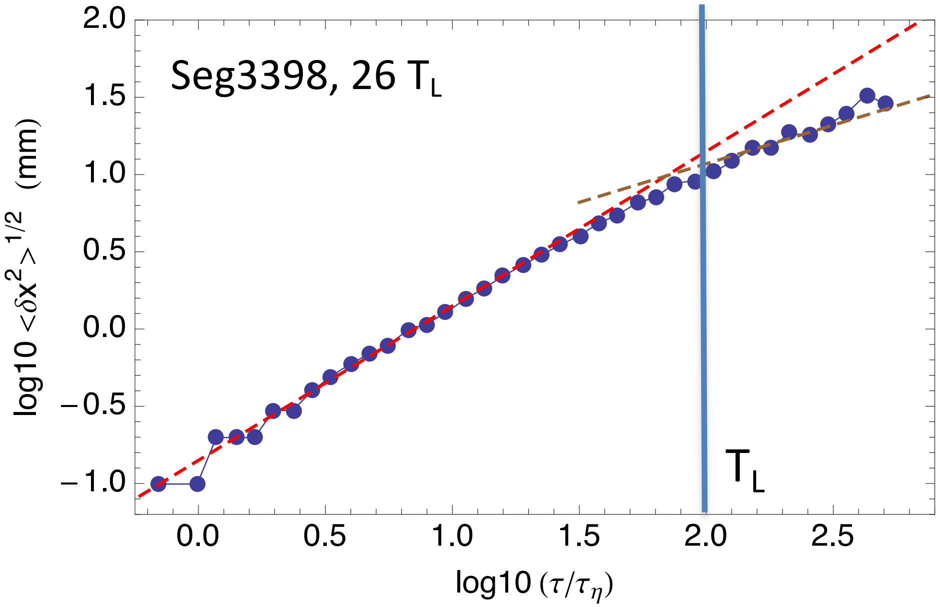
- Application in astrophysics : turbulence of interstellar medium, accretion disks, solar wind …
- Difficulties :
- x Brownian case and incompressibe exact result already 2 coupled Schrodinger eqs
- u Brownian cases …going to u space but now also the b field (not a coordinate) could be for ex Brownian by itself, not only through the coordinate +possible correlations between u and b…
- We have now 2 coupled Brownian motions on (u,b) or on the Elsasser variables (z+,-=u+-b)
- with various subcases , incompressible and compressible…

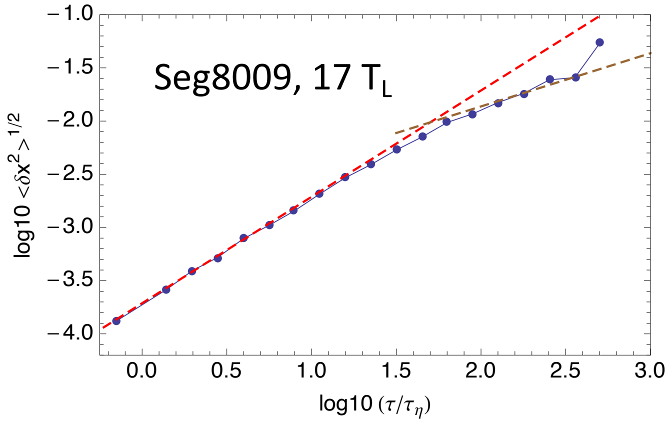
6) Conclusions (with part 5)

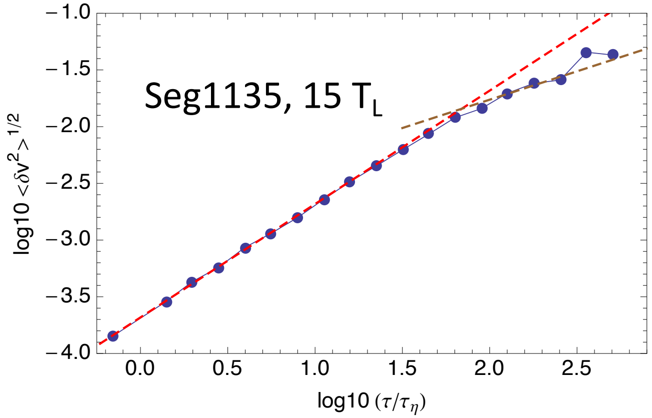
- Extension of previous predictions of SR for more general turbulences than HIT
- New predictions in course for simple rotating turbulence (to be added in particular the shear in v)
- Work in progress for compressible HD and in the MHD (incompressible or compressible) cases
- Numerical simulations….

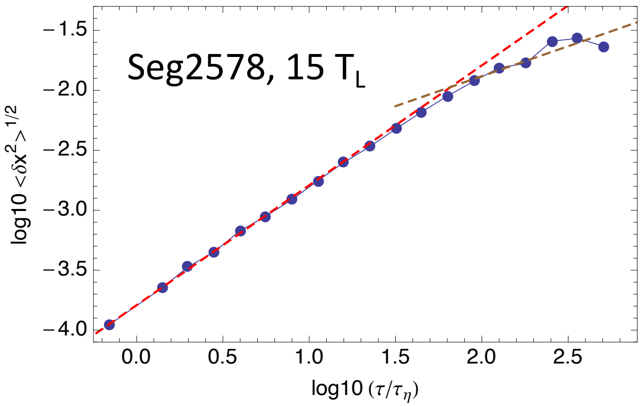
Scaling in x-space ($dx \approx dt^{1/2}$) : other new results

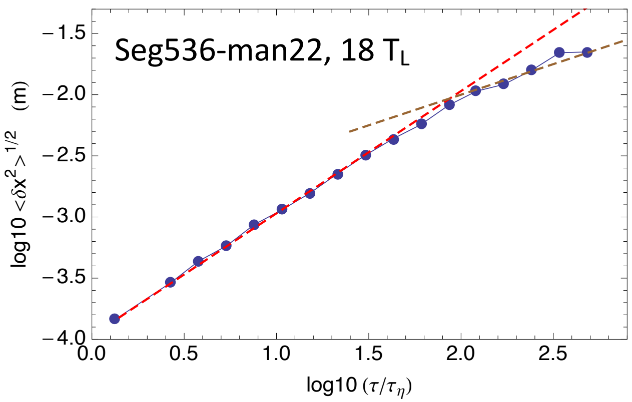
Longest segments of N. Mordant experiment, time-scales >> T_L: BECOME FRACTAL (D_F=2) IN X-SPACE ! -> New transition to fractal space at large scales ?
→Schrödinger-type regime in x-space ?

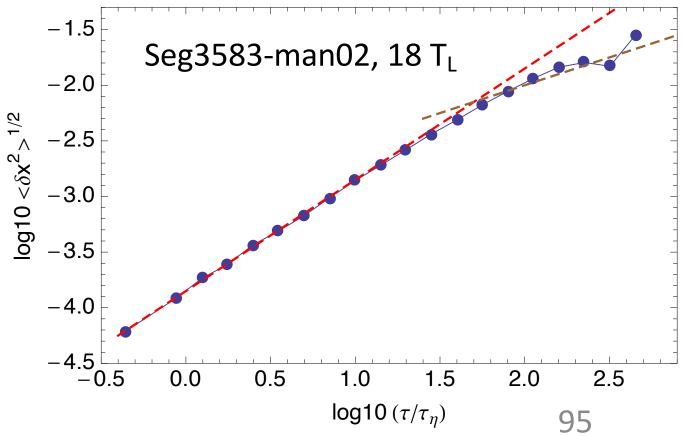












If SR in usual x space :thus change of notations here v->x, a->v Dv->Dx…Arond->Vrond

$$dv = a\,dt + dW \qquad dW = \zeta\sqrt{2D_v dt}$$

$$a \longrightarrow \{a_+, a_-\} \quad \rightarrow \mathcal{A} = \frac{a_+ + a_-}{2} - i\,\frac{a_+ - a_-}{2}$$

(Irreversibility -> doubling of acceleration vector)

$$\mathcal{A} = -2iD_v \nabla_v \ln \psi_v$$

(potential part of acceleration)

$$\frac{\widehat{d}}{dt} = \frac{\partial}{\partial t} + \mathcal{A}.\nabla_v - iD_v\Delta_v$$

(New form of total derivative)

→ Motion equation : Navier-Stokes → (ρ=1,if incompressible)

$$\frac{dv}{dt} = F = -\nabla p + \nu\Delta v$$