La constitution de l'espace physico-mathématique. Vers la biologie.

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There is no space in Euclid's Geometry

A Geometry of figures, handled by ruler and compass,

No mathematics of plane ("apeiron")


MAXIMIANVS
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Lo spazio in Giotto, Scrovegni, Padova, 1300

Lo spazio in Giotto, Assisi, 1297-1300

Le débat metaphysique

Saint Thomas et l'infini actuel de Dieu, au delà d'Aristote

Le problème de **la grâce de Marie**

L'évêque Templier (Paris 1275)

P. Zellini, **A brief history of infinity**. Penguin, 2005 (italien : Adelphi, 1980)

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Le problème de **la grâce de Marie**

L'évêque Templier (Paris 1275)

P. Zellini, **A brief history of infinity**. Penguin, 2005 (italien : Adelphi, 1980)

L'infini et les **annonciations** dans la peinture italienne

Sara Longo, "*L'annonciation en Italie. Enjeux méthodologiques …*", à paraître

G. Longo. "*L'infini mathématique "in prospettiva" et les espaces des possibles*". A paraître (téléchargeable)

Infinity in Mathematics, via Paintings

Early perspective in Italian Renaissance (from *Sara Longo*, **PhD Thesis**):

A. Lorenzetti "***Annunciation***", God vs. Mary, 1344

Masaccio, Beato Angelico …. San Bernardino da Siena (1380-1444) :

the **Annunciation** = ***l'incommesurabile nel misurabile***

Infinity in Mathematics, via Paintings

Infinity “in the painting”: Beato Angelico, San Marco (1400-’55)

(Daniel Arasse, *“Histoires de peintures”, 2004;* Sara Longo, *Thèse Doct., ’13*)

Infinity in Mathematics, via Paintings

Projective Geometry Italian Painting, XV century: Brunelleschi,
L. B. Alberti, *Della Pittura*, 1435: **Infinity "in the painting"**
Piero della Francesca: "***Annunciations***", God vs. Mary, 1470

L’infinito nel quadro: la geometria proiettiva

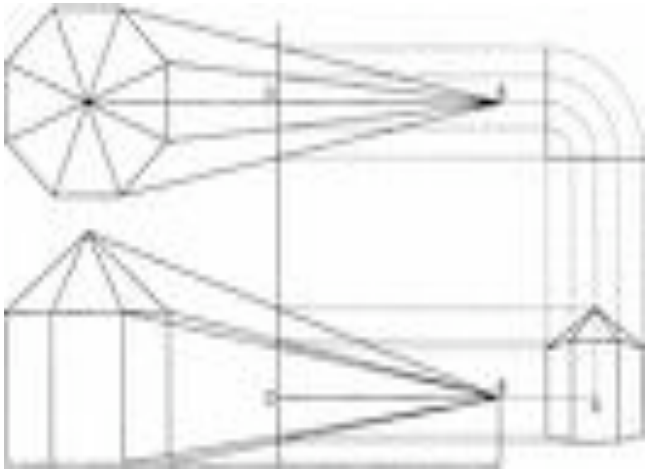
Piero della Francesca, 1466

Infinity in Mathematics, via Paintings

From infinity “in the painting” to the mathematics of infinity:

Piero della Francesca **De perspectiva pingendi** (~1450)

Costruction of a “point of view”:



CRIVELLI Carlo, *Annonciation avec saint Emidius,*
1486, détrempe sur bois, 207 x 146,7 cm.
Londres, National Gallery.

Sara, 13

Wermer (1632-75)

. ... Van Frassen; Copernico ...

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From Physics towards Biology: Phase Spaces and Enablement

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SPACES IN PHYSICS

The construction of **space** in Physics:

First analyze trajectories (Aristotle, Galileo ...), *then*

invent the right **"*spaces*"**

No space in Euclid's geometry,

the mathematical reference for Galileo.

SYMMETRIES AND SPACES IN PHYSICS

The origin of Modern Physics (*step 1)*:

1 – **Descartes spaces:**

The space of all possible trajectories, defined by symmetries

2 - **Galileo's inertia:**

A *momentum conservation* law

1 + 2 : Galileo's **relativity group** (the ***symmetry*** *transformations* that preserve the physical invariants)

PHASE SPACES IN PHYSICS

The construction of **phase space** in Physics *(steep 2)*:

add the invariants that matter:

- **Boltzmann, Poincaré** (1880-90): analyze the trajectory in "position, momentum" spaces (since then the canonical "***phase space***" of physics)
- **Einstein**: after the *invariance* of the speed of light (in its trajectory), use Riemann's geometry in the classical phase space (+time).
- **Thermodynamics**:

  the p, V, T space of the thermodynamic trajectory.

PHASE SPACES AND SYMMETRIES IN PHYSICS

Thus the *phase space* is proposed on the grounds of

- **Symmetries** (beginning with Descartes axes)
- **Invariants of the trajectories** (momentum), thus, again, ***symmetries*** (Noether's theorems)

Similarly for **E**, **t** (as for p, q; all "conjugated variables" in QM)

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"*Put in the "background*" the intended space of trajectories - the pertinent one for the trajectories:

- **parameters** (space, time) *and*
- **invariants** observables (momentum, energy ...)

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Also in **QM:** the "*wave or state function"* goes along a trajectory in **Hilbert Spaces** (Schroedinger's choice of the phase space)

In summary:

- Analysis of trajectories (inertia)
- Equations,
- Symmetries (invariants)
- Phase space

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Causes become interactions and these interactions themselves constitute the fabric of the universe of their manifestations, its geometry: modifying this fabric changes the interactions; changing the interactions modifies the fabric. (Bailly, Longo, ch.3)

So far for the ***"structure of determination"***

in physics …

A few words on Time

Time in the “geometric” vs “algebraic-formal” approaches to Mathematics

In the “**geometric**” approach: time is the ***time of genesis of structures***, the recording medium of their *process of constitution*.

In the “**algebraic-formal**” approach: time is a matter of sequential functioning, of the *execution of algorithms*. (Bailly, Longo, ch. 3)

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In *the Foundations of Mathematics*, difference between:

- **principles of construction** (in particular those with a geometrical nature, symétries and order principles)

vs.

- **principles of proof** (formal principles of logical deduction).

Mathematics is built up on the basis of both types of principles. (Bailly, Longo, introduction and ch. 1)

ENABLEMENT in BIOLOGY

Longo G., Montévil M., Kauffman S. *No entailing laws, but enablement in the evolution of the biosphere*.

GECCO'12, July, 2012, Philadelphia (PA, USA); proceedings, 2012.

Longo G., Montévil M. *Extended Criticality, Phase Spaces and Enablement in Biology.*

Special Issue of **Chaos, Solitons and Fractals**, 2013.

RECALL FROM PHYSICS

Galileo, Newton: **“Causal” relations**: e. g.

$$f = ma$$

Since E. Noether, H. Weyl, … van Fraasen

Geodetics and **symmetries**

in the right phase spaces

A unifying (better) frame for intelligibility of “causes”.

FROM PHYSICS TO BIOLOGY

Biological analysis, since Darwin, use proper observables:
organisms (and *phenotypes*).

E. g.: Thermodynamics: entropy (and its proper principle)…

Hydrodynamics, QM … later unify …

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Hydrodynamics, QM … later unify …

Note: empirical evidence for *causes*:

differences *causing* differences

E. g. identical acceleration *implies* identical force (Galileo's gravitation); yet, the "general law" is not evident (Newton).

FROM PHYSICS TO BIOLOGY: ENABLEMENT

Gravitation *causes* a body to fall

Gravitation **is not** a *cause* in biology: it is a constraint,

It contributes to the evo/devo in a niche/ecosystem.

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Key reason:

The **default state** in physics is **inertia**

The **default state** in biology is "**proliferation with variation**" (Darwin's first principle) *and* **motility.**

Changing constraints (the formation of a new niche)

Enables a variation to succeed

A omnipresent phenomenon in evolution: **allopatric speciation** (a species formed from a population in a different niche/ecosystem)

ENABLEMENT and CAUSES

Claim: (evident) causes are differential in biology

A (a-causal/random) mutation (a difference) may ***cause*** a phenotypic difference.

Bacteria (a difference from normality) *may cause* a pneumonia.

Yet, this causal consequence **may be *enabled* by the niche** (a wounded lung, a weak immune system ..)

A different role of *constraints*, in physics vs. biology due to the difference in the default states:

inertia vs. proliferation (with variation)

An organism does not need a "cause" to be active (to move and proliferate): selection inhibits action.

ENABLEMENT and CAUSES

Different role of *constraints*, in physics vs. biology

Examples:

1 - **A river at a bifurcation**: fully deterministic analysis (possible highly non linear, thus unpredictable) of inert (gravitational) matter, in a given space of observables (energy, momentum…).

2 - **A population proliferation facing a "niche multifurcation"**: phenotypic variations are co-constitued with the possible enablement(s) by one or more possible evolutionary paths (a different default state)

In 1, the structure of determination *describes causes*;

in 2, it must *include enablement*.

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in 2, it must *include enablement*.

Physical objects never go wrong.

ENABLEMENT and PHASE SPACES

Enablement "**enables**" new organisms and phenotypes:

these are the proper observables

Darwinian frame: Jacob's bricolage, Gould's exaptation (cavefish, lung-fish's bladder, double-jaw …)

On top of the physical phase space: **add** biological observables, the phenotypes.

Compatible **extensions** of physical theories.

The issue of **randomness**: different notions in physics ….

RANDOMNESS and PROBABILITIES

Biological randomness is “moved up”:

It is not within a pre-given phase space like in physics

(from the 6 values of classical dice, to quantum measurement),

The very **phase space is randomly generated** by the dynamics of evolution (ontogenesis).

The **impossibility to “measure” biological randomness by probabilities**: probability is a measure between considered case(s) and a given space of possibilities.

Which were the probability for Allucinoginea to survive?

And the {tetrapod} to develop in a elephant or a squirrel?

AUTONOMY and CONSTRAINTS

Life is an entangled blend of **autonomy** *and* **constraints**.

Autonomy (the Kantian whole/ autopoiesis) is possible only *under ecosystemic (and autopoietic internal) constraints.*

Constraints have *a biological meaning* only because of the autonomy of organisms (because they interact with organisms).

An organism can "*stand*" (*adapt to*) a slightly, yet ever changing ecosystem by its autonomy: it has a (relative) biological inertia.

An **ever changing inertial dynamics** as based on *internal proliferation with variation*, essential to the autopoietic reconstruction.

NON-ERGODICITY and CONVERGENCE

Non-ergodicity:

Evolution does not explore all possible molecular combinations:

Impossible to produce all proteins length 200 amino acids in 10^39 times the lifetime of the universe, even were all 10^80 particles making such proteins on the Planck time scale.

Randomness *and* **constraints**:

History, at the level of phenotypes, **canalizes** (sets constraints to) the exploration of new phenotypes.

Similar functional constraints, possible interpretation for:

- The convergence of the vertebrate and pulp's eye;
- The anologies in the tasmanian (marsupial) and the mammalian wolf.

Some references

http://www.di.ens.fr/users/longo *or* **Google: Giuseppe Longo**

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