On the measure process between different scales

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Abstract
In this communication, we follow the fractaquantum hypothesis and try to generalize the framework of quantum mechanics to macroscopic scales. We propose a tentative to set the fundamental problem of measuring process done by a large structure on a microscopic one. We consider the example of voting when an entire society tries to measure globally opinions of all social actors in order to elect a delegate. We propose a quantum model to interpret an operational voting system.

Key words: Fractaquantum hypothesis.

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1 Introduction

• Matter is constituted by discrete quanta (that we call classical atoms in this contribution) and this fact was empirically put in evidence by E. Rutherford in the beginning of 20th century. Light is also discretized into quanta, as demonstrated by the photoelectric effect discovered by H. Hertz at the end of 19th century and explained by A. Einstein in 1905 with the photon hypothesis. A major consequence of these discoveries for our common life in 21th century is the existence of lasers, transistors and computers.

• The stability of classical atoms is not understandable in the framework of classical mechanics and electromagnetism. Thus quantum mechanics was developed in the 1930's to explain this stability; the names of N. Bohr, M. Born, L. De Broglie, W. Heisenberg, E. Shrödinger, are strongly associated with the discoveries done during this period. The result is a mathematical formalism described into details *e.g.* in the book of C. Cohen-Tannoudji *et al* [4]. A recent reflexion of M. Mugur-Schächter [11] points clearly the fact that microscopic quanta as classical atoms or photons are not directly perceptible by our senses. In consequence, any possible knowledge for a human observer of a microscopic quantum is founded on the experimental protocols. The notion of what a scientist call "experiment" has been to be re-considered. The interaction between a microscopic quantum and the measuring apparatus changes the element of Nature that is observed. In some sense, an *a priori* or an external description of Nature is not possible at quantum scale. The philosophical consequences of this new vision of Nature are still under construction and we refer to B. D'Espagnat [6], M. Bitbol [2], B. Nicolescu [12] among others.

• Independently of the development of this renewed physics, the importance of scale invariance have been recognized by various authors as B. Mandelbrot [10] and L. Nottale [13]. The word "fractal" is devoted to figures and properties that are self-similar whatever the refering scale.

• We have suggested in 2002 the fractaquantum hypothesis [7], founded on two remarks: Nature develop a scale invariance and quantum mechanics is completely relevant for small scales. In order to express this hypothesis, we have introduced [8] the notion of "atom", in fact very similar to the way of vision of Democrite and the ancient Greek philosophers (see *e.g.* J. Salem [14]). To fix the ideas, an "atom" can be a classical atom, or its nucleus, or a molecule, or a micro-organism like a cell, or an entire macro-organism as a human being or till an entire society! If we divide an "atom" into two parts, its qualitative properties change strongly at least in one of these parts [9]. The framework of fractaquantum hypothesis suggests that formulation of quantum mechanics can be applied to all "atoms" in Nature, whatever their size. According to the *Infra quantum mechanics* [11], a "micro-state" relative to a human observer is an "hypothetical entity that no human can detect". In this particular case, a ℓittle "atom" ℓ is a classical atom and a big "atom" B is a human observer. More generally, two "atoms" ℓ and B have different scales when "atom" ℓ is not directly perceptible to "atom" B. In other words, a direct interaction between B and ℓ can not be controlled by B himself. In this case, the direct interaction between little "atom" ℓ and big "atom" B can be neglected as a first order approximation.

• In these conditions, the fractaquantum hypothesis suggests that the measure process of some characteristics of "atom" ℓ follows the mathematical framework of quantum mechanics [4]. The "atom" ℓ is modelized mathematically by a vector also denoted by ℓ in an Hilbert space H of configuration. The action of measurement supposes the existence of a self-adjoint operator A. This operator is completely determined by the macro "atom" B which chooses the physical quantity to measure and by the rules of quantification. The result of this process is necessarily an eigenvalue α of this operator A and due to the measure interaction, the "atom" ℓ is projected onto the corresponding eigenspace denoted typically by E_α. Moreover, the Born rule claims that the probability of observing the datum α as a result of the mesurement is given by the norm of the projection of ℓ on the eigenspace E_α. We refer the reader *e.g.* to C. Cohen-Tannoudji *et al* [4] and M. Mugur-Schächter [11] for a detailed description and motivation of this mathematical formalism.

• In this contribution, we revisit this classical quantum formalism when little and big "atoms" are nonclassical ones. In fact, this research program is tremendous! The phenomenology of possible measurement interactions should be reconstructed. What is a big "atom" B that can measure some quantities on little "atom" ℓ? Does the classical framework of quantum mechanics operates without any modification? Of course all these questions motivate our communication. Due to the lack of knowledge of what can be a measure done by "atoms" at mesoscopic or microscopic scales, we restrict ourselves to particular cases of measures done by human beings about microscopic atoms and to measures done by human society considered as a whole on individual human beings. Precisely, we consider here only a very particular example: the measurement process associated with voting. In this case, "atom" ℓ is a social actor and "atom" B is the entire society. We propose to model a voting process with the help of classical rules of quantum mechanics.

2 An election as a quantum measurement process

• We consider a macroscopic "atom" B composed by an entire social structure. For example, B is a state like France to fix the ideas. The social actors of society B are the little "atoms" ℓ in our model. We write here

$$\ell \in \mathrm{B} \tag{1}$$

even if this expression (1) is not completely correct from a mathematical point of view. The numbers of such indistinguable individuals are quite important (10^6 to 10^9 typically).

The democratic life in society B supposes that social responsabilities are taken by elected representants of social corpus. Thus a voting process has the objective to determine one particular social actor among all for accepting social responsabilities. This kind of position is supposed to be attractive and a set Γ of candidates γ among the entire set of "atoms" ℓ is supposed to be given in our framework.

• The problem is to determine a single "elected" candidate γ_1 among the family Γ thanks to the synthesis of all opinions of different electors ℓ. The social objective of society B is the determination of one candidate among others through a social process managed by the entire society, modelized here as a macro "atom" B. This problem is highly ill posed and we refer to the pioneering works of J.C. de Borda [3] and N. Condorcet [5] followed more recently by the theorem of non existence of a social wellfare function satisfying reasonable hypotheses, proved by K. Arrow [1]. Nevertheless, we restrict here to the so-called "first tour" process as implemented in a lot of situations. In this process, each elector ℓ has to transmit the name of at most one candidate γ. Then an ordered list of candidates is obtained by counting the number of expressed votes for each candidate.

• A quantum model of such a process is possible. Introduce the space H_Γ of candidates generated formaly by the finite family Γ of all candidates:

$$H_\Gamma \,=\, \bigoplus_{\gamma \in \Gamma} \mathbb{C}\, \gamma \qquad (2)$$

where $\mathbb{C}$ denotes the field of complex numbers. This decomposition (2) is supposed to be orthogonal:

$$(\gamma ,\, \gamma') \,=\, \left\{ \begin{array}{ll} 0 \quad \text{if} & \gamma \neq \gamma' \\ 1 \quad \text{if} & \gamma = \gamma', \end{array} \right. \,, \qquad \gamma, \gamma' \in \Gamma .$$

The "wave function" associated with an elector ℓ is represented by a state also denoted by ℓ in this space H_Γ:

$$\ell \,=\, \sum_{\gamma \in \Gamma} (\ell ,\, \gamma)\, \gamma \,. \qquad (3)$$

The scalar product $(\ell ,\, \gamma)$ in relation (3) is the component of elector ℓ relative to each candidate γ. This number represents the political sympathy of elector ℓ relatively to the candidate γ. We suppose here that the norm $\|\, \ell \,\|$ of state ℓ *id est*

$$\|\, \ell \,\| = \sqrt{\sum_{\gamma \in \Gamma} |\, (\ell ,\, \gamma)\, |^2}$$

is **inferior or equal** to unity. We follow the Born rule and suggest that the probability for elector ℓ to give its vote to candidate γ is equal to $|\, (\ell ,\, \gamma)\, |^2$. We suggest also that the probability to answer by a vote "blank or null" is $1 - \|\, \ell \,\|^2$ in this framework.

• The interpretation of the projection process in the quantum measurement for such a first tour of election process is quite clear. During the election, *id est* the particular day where the measure process occurs, the elector ℓ is obliged to choose at most one candidate γ_0. In consequence, all his political sensibility is socially "reduced" to this particular candidate. We can write:

$$\ell = \gamma_0$$

to express the wave function collapse. This quantum interpretation of such voting process clearly shows the violence of such kind of decision making. Of course, no elector has political opinions that are identical to one precise candidate and this measurement process is a true mathematical projection. Nevertheless, the social voting process imposes this projection in order to construct a social choice. The disadvantage and dangers of such process have been demonstrated in France during the presidential election process in 2002 (see *e.g.* [15]).

3 Conclusion

• In the framework of fractaquantum hypothesis, the mathematical formalism of quantum mechanics is supposed to have an extension to all "atoms" in Nature, whatever their size. In particular, the process of measuring has to be re-visited to all pairs (ℓ, B) of "atoms" with different scales. With the example of a classical election, the large scale imposes a direct generalization of the measure process in quantum mechanics and all the characteristics of the mathematical measure operator are controled by the large scale. We have noticed the violence of a multiscale interaction through such a measuring process.

• We insist to finish on the notion of scale difference introduced in the title of this contribution. Two "atoms" ℓ and B have different scales when "atom" ℓ is not directly perceptible to "atom" B. The perception, *id est* the consciousness of direct interaction between a little "atom" ℓ and a big one B is neglected when ℓ and B have different scales. In consequence, the notion of perception between two "atoms" should be precisely defined in all generality in future works.

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