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An analytical formalism of dynamical systems

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Abstract

A *dynamical system* is considered as a system of coupled first order dynamic differential equations in this paper. A system like this represents the set of mathematical relationships among the dynamics of different variables, i.e. the dynamic structure of the system. In physics, natural laws are represented by systems of coupled second order dynamic differential equations, and the dynamic structure is given by Newton's laws.

However, natural laws in physics can be deduced from the minimization of integral action minimum principle, which through the Euler-Lagrange equations provides a Lagrangian-Hamiltonian formalism of natural laws. This action principle provides a teleological approach to natural laws besides the mechanical approach provided by Newton's laws (the dynamic structure).

Can an integral action minimum principle be applied to a dynamical system, i.e, by a teleological approach? The answer is positive. The combination of two independent theories can be applied to get this new formalism: the generalized Hamiltonian dynamics developed by P. A. M. Dirac in 1950, and the inverse Lagrange problem for dynamical systems developed by P. Havas in 1957. Particularly, this formalism also permits a quantum formulation of dynamical systems presented in a subsequent paper of this congress (*a quantum formalism of dynamical systems*). This approach is here called as the *analytical formalism of dynamical systems*.

First of all, the analytical formalism permits to state a mathematical parallelism between dynamical systems and physical systems. Besides, it permits the definition of functions such as the Lagrangian function or the Hamiltonian function of a dynamical system, and getting particular conserved amounts such as the energy (in the autonomous case) that a dynamic structure approach does not permit. In addition, other fundamental equations can be obtained, such as the Hamilton-Jacobi equation.

In general, the analytical formalism of dynamical systems presented is a tool to explore deeper the dynamics of complex systems, in addition to provide a way to define a quantum approach to such systems.

An application case is presented: the case of the one-dimensional systems, concretely the logistic function case. For this case the Lagrangian and Hamiltonian functions are obtained, as well as the system energy. The discussion about future lines of investigation and the link with the quantum approach is done to end the paper.