A MACROSCOPIC SPATIO-TEMPORAL MODEL OF THE BRAIN

Symposium 11: Mathematical modeling of complex systems

Authors: Joan C. Micó, A. Caselles, S. Amigó, Maria T. Sanz.

1. The time response model.
2. The spatio-temporal response model.
3. The analytical solution of the spatio-temporal response model.
4. Hypotheses about how to validate the spatio-temporal response model.

1. The time response model.

(See a deeper presentation in "*The Unique Personality Trait Theory: a general system theory of human personality*", Symposium 6)

$$\left.\begin{array}{c}\dfrac{dy(t)}{dt} = -a \cdot y(t) + \sum_i p_i \cdot s_i(t) - \sum_i q_i \cdot \int_0^t e^{\frac{x-t}{\tau_i}} \cdot s_i(x) \cdot y(x) \cdot dx \\ y(0) = y_0\end{array}\right\}$$

y(t): brain time activity

y_0: Initial brain activation level dynamics

$s_i(t)$: Stimuli, i=1, 2,…, N

$-a \cdot y(t)$: Homeostatic control

$p_i \cdot s_i(t)$: Excitation effects, i=1, 2,…, N

$q_i \cdot \int_0^t e^{\frac{x-t}{\tau_i}} \cdot s_i(x) \cdot y(x) \cdot dx$: Inhibitor effects, i=1, 2,…, N

2. The spatio-temporal response model: hypotheses

$y(t) \rightarrow \Psi(t, \boldsymbol{r})$: brain spatio-temporal activity

$y_0 \rightarrow \Psi_0(\boldsymbol{r})$: Initial brain spatio-temporal activity

$\boldsymbol{r} = (x_1, x_2, x_3) \rightarrow$ The three spatial coordinates in brain

$$y(t) = \iiint_{\partial V} \Psi(t, \boldsymbol{r}) d\boldsymbol{r}$$

∂V: mathematical brain boundaries

Addition of a diffusion spatial dynamic term:

$$\sigma \cdot \nabla^2 \Psi(t, \boldsymbol{r})$$

$\sigma \rightarrow$ Diffusion Coefficient

2. The spatio-temporal response model: equation

$$\frac{\partial \Psi(t,\boldsymbol{r})}{\partial t} =$$

$$= -a \cdot \Psi(t,\boldsymbol{r}) + \sum_i p_i \cdot s_i(t) \; -$$

$$- \sum_i q_i \cdot \int_0^t \mathrm{e}^{\frac{x-t}{\tau_i}} \cdot s_i(x) \cdot \Psi(x,\boldsymbol{r}) \cdot dx +$$

$$+ \sigma \cdot \nabla^2 \Psi(t,\boldsymbol{r})$$

Initial brain spatio-temporal activity → $\Psi(0,\boldsymbol{r}) = \Psi_{\boldsymbol{0}}$

&

Boundary conditions in the limits brain ∂V : depending on the brain geometry

3. The analytical solution of the spatio-temporal response model.

$$\Psi(t,\boldsymbol{r}) = \rho(t) + \Phi(t,\boldsymbol{r})$$

$$\downarrow$$

$$\rho'(t) + a \cdot \rho(t) + \sum_i q_i \int_0^t e^{\frac{x-t}{\tau_i}} \cdot s_i(x) \cdot \rho(x) \cdot dx =$$

$$= \sum_i p_i \cdot s_i(t)$$

&

$$\frac{\partial \Phi(t,\boldsymbol{r})}{\partial \mathrm{t}} =$$

$$= -a \cdot \Phi(t,\boldsymbol{r}) - \sum_i q_i \int_0^t e^{\frac{x-t}{\tau_i}} \cdot s_i(x) \cdot \Phi(x,\boldsymbol{r}) \cdot dx +$$

$$+ \sigma \cdot \nabla^2 \Phi(t,\boldsymbol{r})$$

3. The analytical solution of the spatio-temporal response model.

$$\Phi(t, \boldsymbol{r}) = \eta(t) \cdot \Omega(\boldsymbol{r})$$

$$\downarrow \qquad (\xi\text{: separation constant})$$

$$\eta'(\mathrm{t}) + (a - \xi) \cdot \eta(\mathrm{t}) + \sum_i q_i \int_0^t \mathrm{e}^{\frac{x-t}{\tau_i}} \cdot s_i(x) \cdot \eta(x) \cdot dx = 0$$

&

$$\nabla^2 \Omega(\boldsymbol{r}) = \frac{\xi}{\sigma} \Omega(\boldsymbol{r}) \quad \text{(Helmoltz equation)}$$

$$\downarrow$$

$$\Psi(t, \boldsymbol{r}) = \rho(t) + \eta(t) \cdot \Omega(\boldsymbol{r})$$

3. The analytical solution of the spatio-temporal response model

Brain geometry: half-sphere → $(r,\theta,\varphi) \in [0,R] \times \left[0,\frac{\pi}{2}\right] \times [0,2\pi]$

$$\left.\begin{aligned} x_1 &= r \cdot sin\theta \cdot cos\varphi \\ x_2 &= r \cdot sin\theta \cdot sin\varphi \\ x_3 &= r \cdot cos\theta \end{aligned}\right\}$$

↓

$$\frac{1}{r^2}\frac{\partial}{\partial \mathrm{r}}\left(r^2\frac{\partial\Omega(\boldsymbol{r})}{\partial \mathrm{r}}\right) + \frac{1}{r^2 sin\vartheta}\frac{\partial}{\partial\theta}\left(sin\vartheta\frac{\partial\Omega(\boldsymbol{r})}{\partial\theta}\right) + \frac{1}{r^2 sin\vartheta}\frac{\partial^2\Omega(\boldsymbol{r})}{\partial\varphi^2} = \frac{\xi}{\sigma}\Omega(\boldsymbol{r})$$

(Helmoltz equation in spherical coordinates)

(Solution) ↓ (Separation constant: $k^2 = -\frac{\xi}{\sigma}$)

$$\Omega_{(l)}(\boldsymbol{r}) = \frac{1}{\sqrt{r}} J_{l+1/2}(k \cdot r) P_l(cos\theta)$$

Quantum number: l=0, 1, 2,…, +∞

P_l: Legendre polynomials

$J_{l+1/2}$: first order Bessel functions to $l + 1/2$ values

3. The analytical solution of the spatio-temporal response model: boundary conditions

$$\left.\frac{\partial \Psi(t,\boldsymbol{r})}{\partial r}\right|_{r=R} = 0 \rightarrow \left.\frac{\partial \Omega(t,\boldsymbol{r})}{\partial r}\right|_{r=R} = 0 \rightarrow 2\mu \cdot J'_{l+1/2}(\mu) - J_{l+\frac{1}{2}}(\mu) = 0\,; \quad \mu = k \cdot R$$

$$\downarrow$$

$$\xi_{n_l} = -k_{n_l}^2 \cdot \sigma$$

Two quantum numbers:
l=0, 1, 2,…, +∞ and for each l: n_l = 1, 2, 3…,+∞

$$\downarrow$$

$$\Omega_{(n_l,l)}(\boldsymbol{r}) = \eta_{n_l}(t)\frac{1}{\sqrt{r}}J_{l+1/2}\left(k_{n_l} \cdot r\right)P_l(cos\theta)$$

$$\downarrow$$

$$\Psi(t,\boldsymbol{r}) = \rho(t) + \sum_{n_{l=1}}^{\infty}\sum_{l=0}^{\infty} \mathrm{A}_{(n_l,l)}\eta_{n_l}(t)\frac{1}{\sqrt{r}}J_{l+1/2}\left(k_{n_l} \cdot r\right)P_l(cos\theta)$$

3. The analytical solution of the spatio-temporal response model: initial conditions

$$\Psi(t,\boldsymbol{r}) = \rho(t) + \sum_{n_{l=1}}^{\infty}\sum_{l=0}^{\infty} \mathrm{A}_{(n_l,l)}\eta_{n_l}(t)\frac{1}{\sqrt{r}}J_{l+1/2}\left(k_{n_l}\cdot r\right)P_l(cos\theta)$$

$$\downarrow \quad \Psi(0,\boldsymbol{r}) = \Psi_0(\boldsymbol{r})$$

$$\Psi_0(\boldsymbol{r}) = \rho(0) + \sum_{n_{l=1}}^{\infty}\sum_{l=0}^{\infty} \mathrm{A}_{(n_l,l)}\eta_{n_l}(0)\frac{1}{\sqrt{r}}J_{l+1/2}\left(k_{n_l}\cdot r\right)P_l(cos\theta)$$

$$\downarrow$$

After some complex calculations

$\mathrm{A}_{(n_l,l)}$ are obtained depending on the initial conditions

(see Equations 25 to 29 in the paper)

4. Hypotheses about how to validate the spatio-temporal response model.

Experimental design 1: Measure by **Medical Image** the spatio-temporal activity $\Psi_0(\boldsymbol{r})$ at an initial time and the spatio-temporal activity $\Psi(t, \boldsymbol{r})$ at different times and compare them with the model outcomes.

Experimental design 2: Measure by **Electroencelography** the spatio-temporal activity $\Psi_0(\boldsymbol{r})$ at an initial time and the spatio-temporal activity $\Psi(t, \boldsymbol{r})$ at different times and compare them with the model outcomes.

Experimental design 3: Compare the brain frequencies with the theoretical frequencies of the model associated with the quantum numbers.