

CNAM, Paris, 4 Octobre 2010

FRACTALS

Des objets fractals
aux espaces-temps fractals

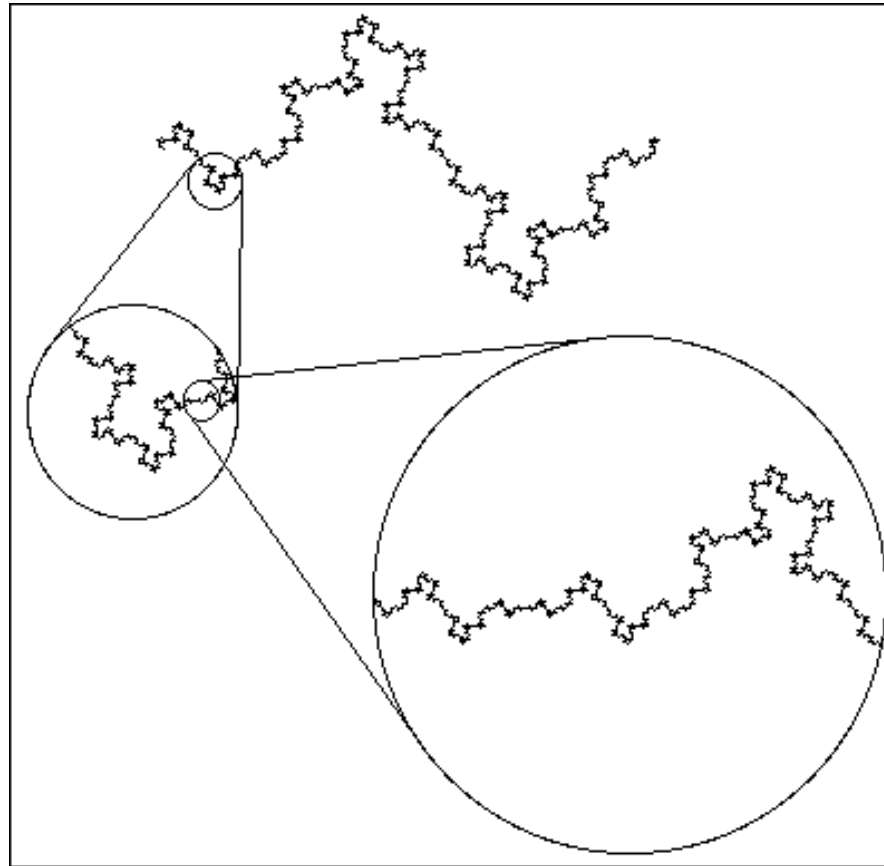
Laurent Nottale

CNRS

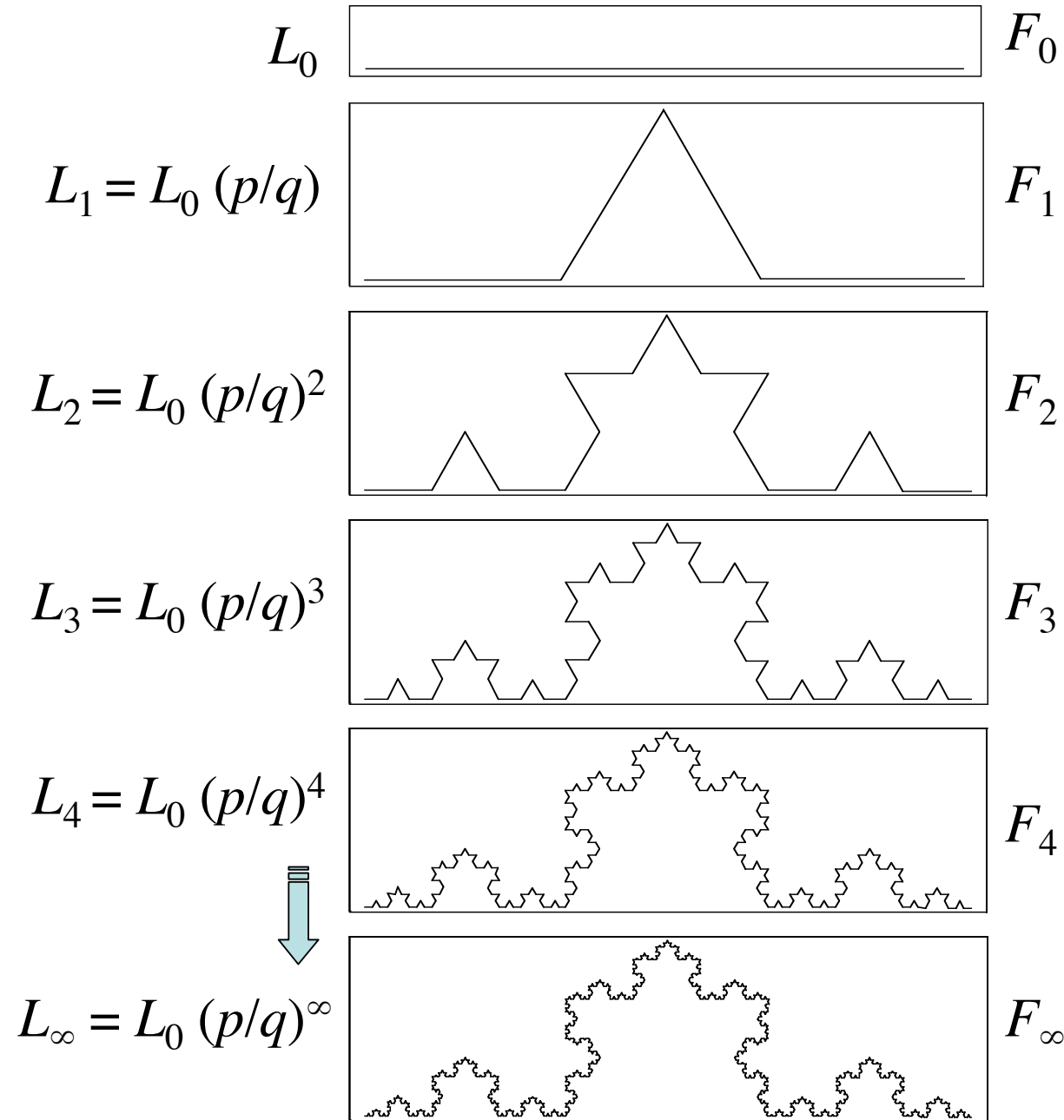
LUTH, Observatoire de Paris-Meudon

Refs: <http://www.luth.obspm.fr/~luthier/nottale/>

Zooms discrets sur une fractale



Courbe de von Koch



Générateur:

$$p = 4$$

$$q = 3$$

Dimension fractale:

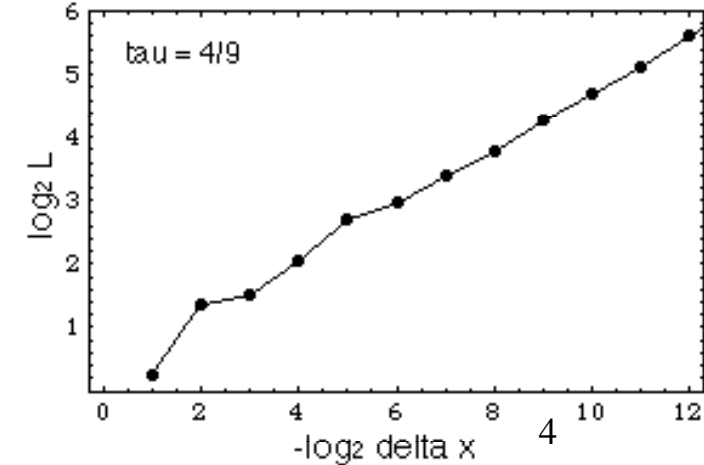
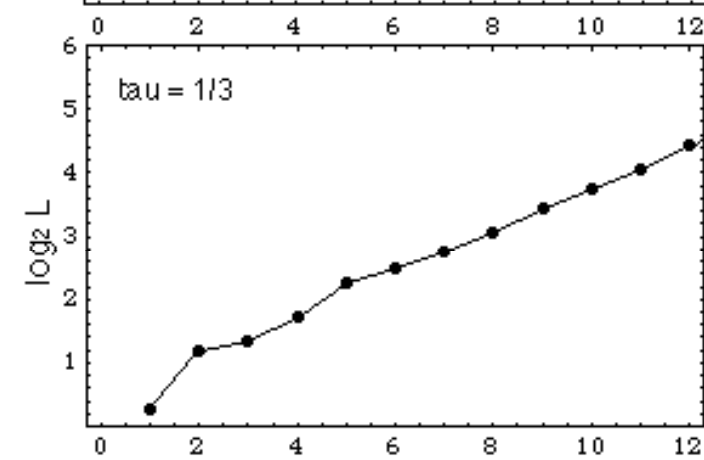
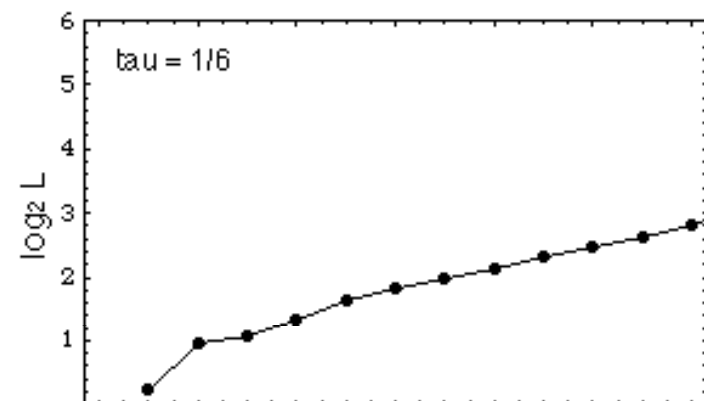
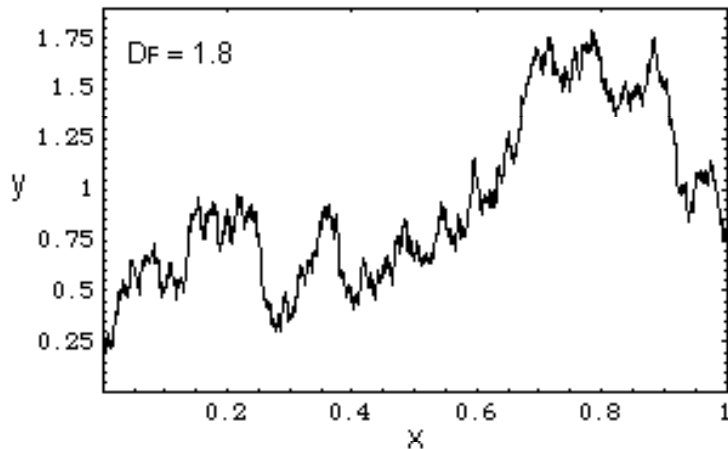
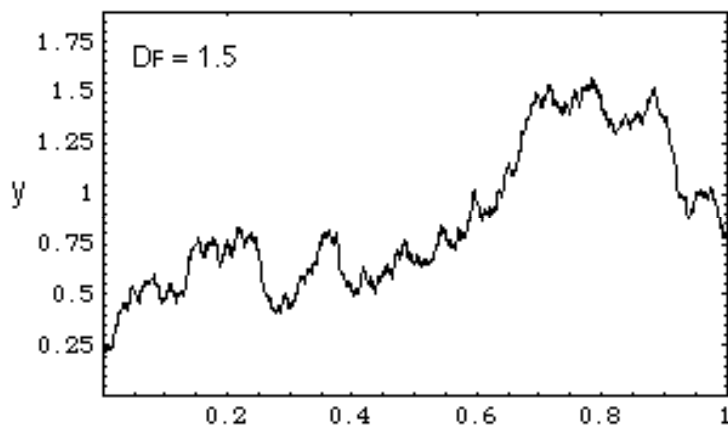
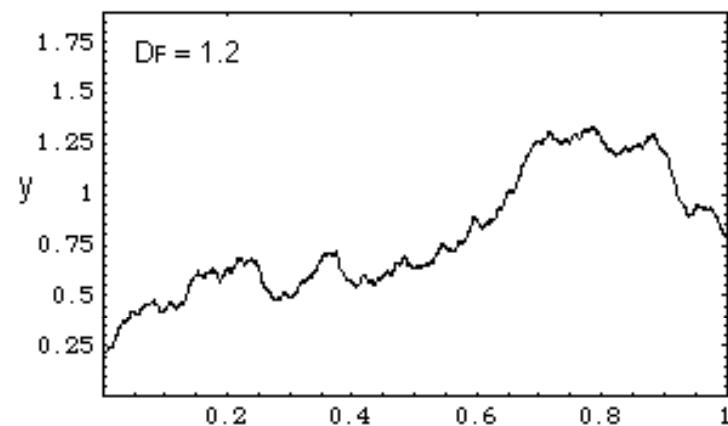
$$D_F = \frac{\log p}{\log q}$$

$$\varepsilon = \lambda \times q^{-n}$$

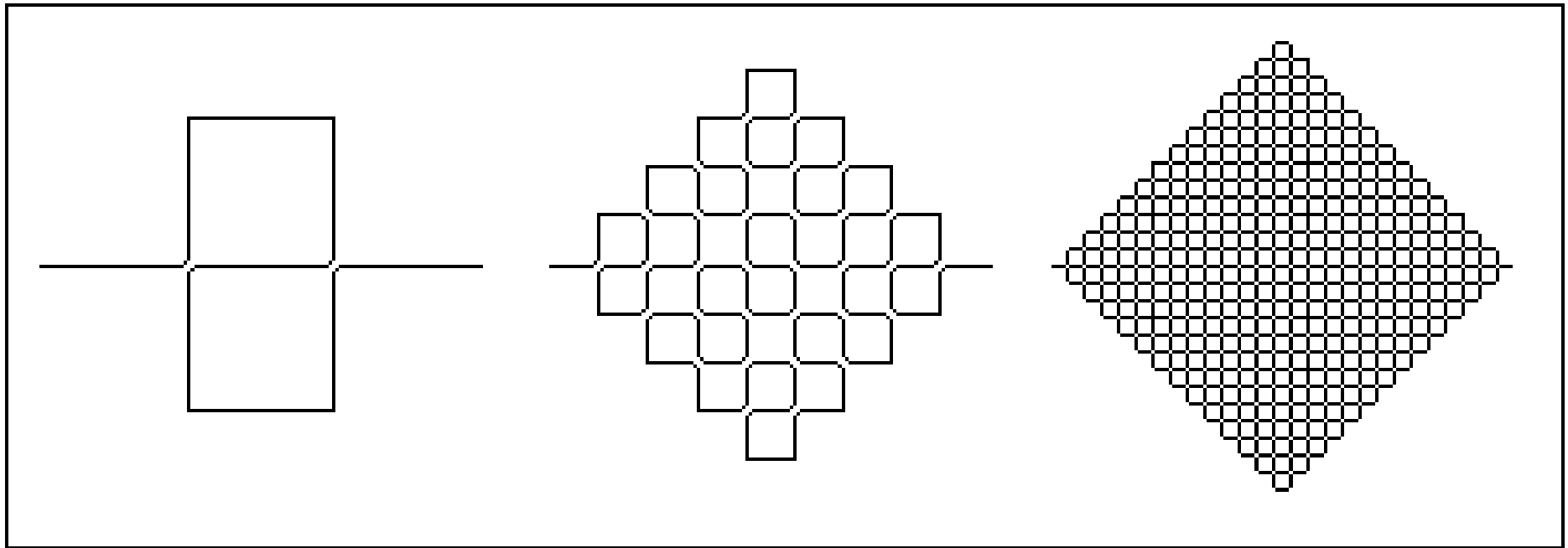


$$L_n = L_0 \left(\frac{\lambda}{\varepsilon} \right)^{D_F - 1}$$

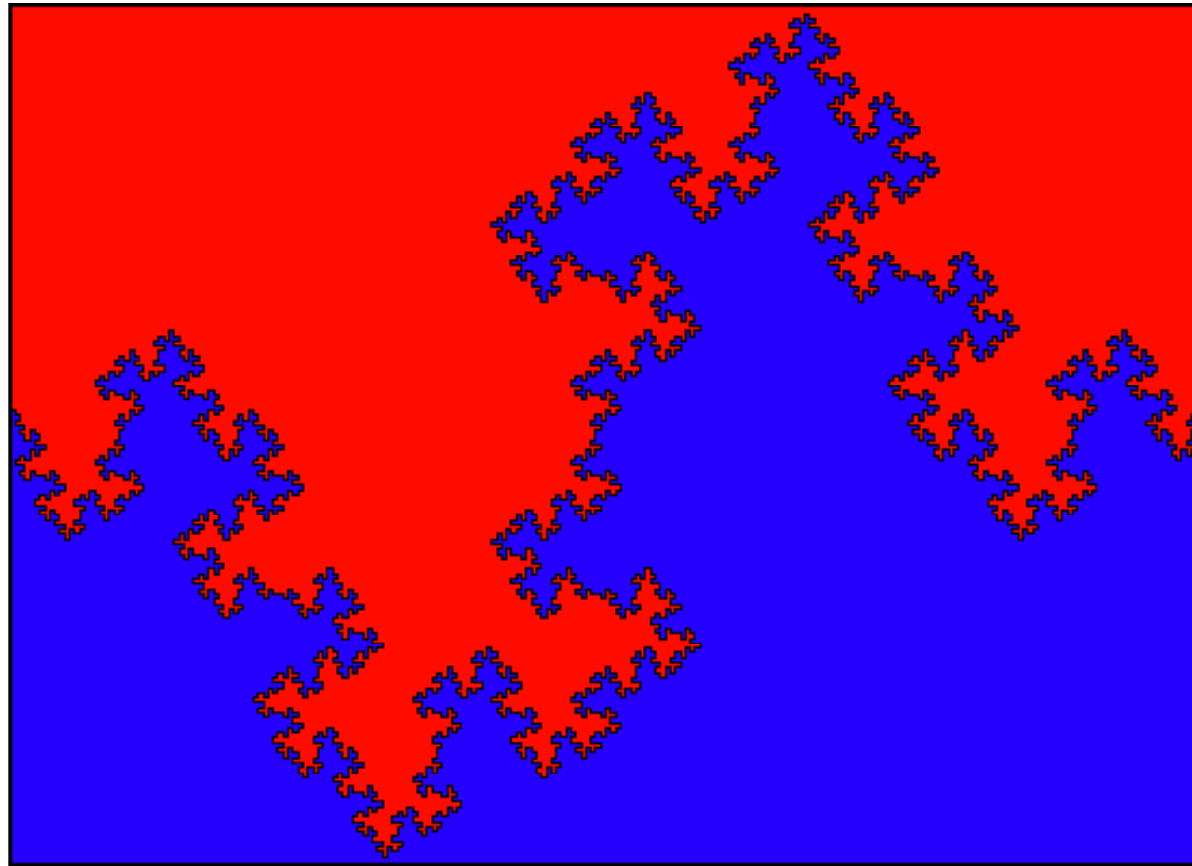
Construction par dissections



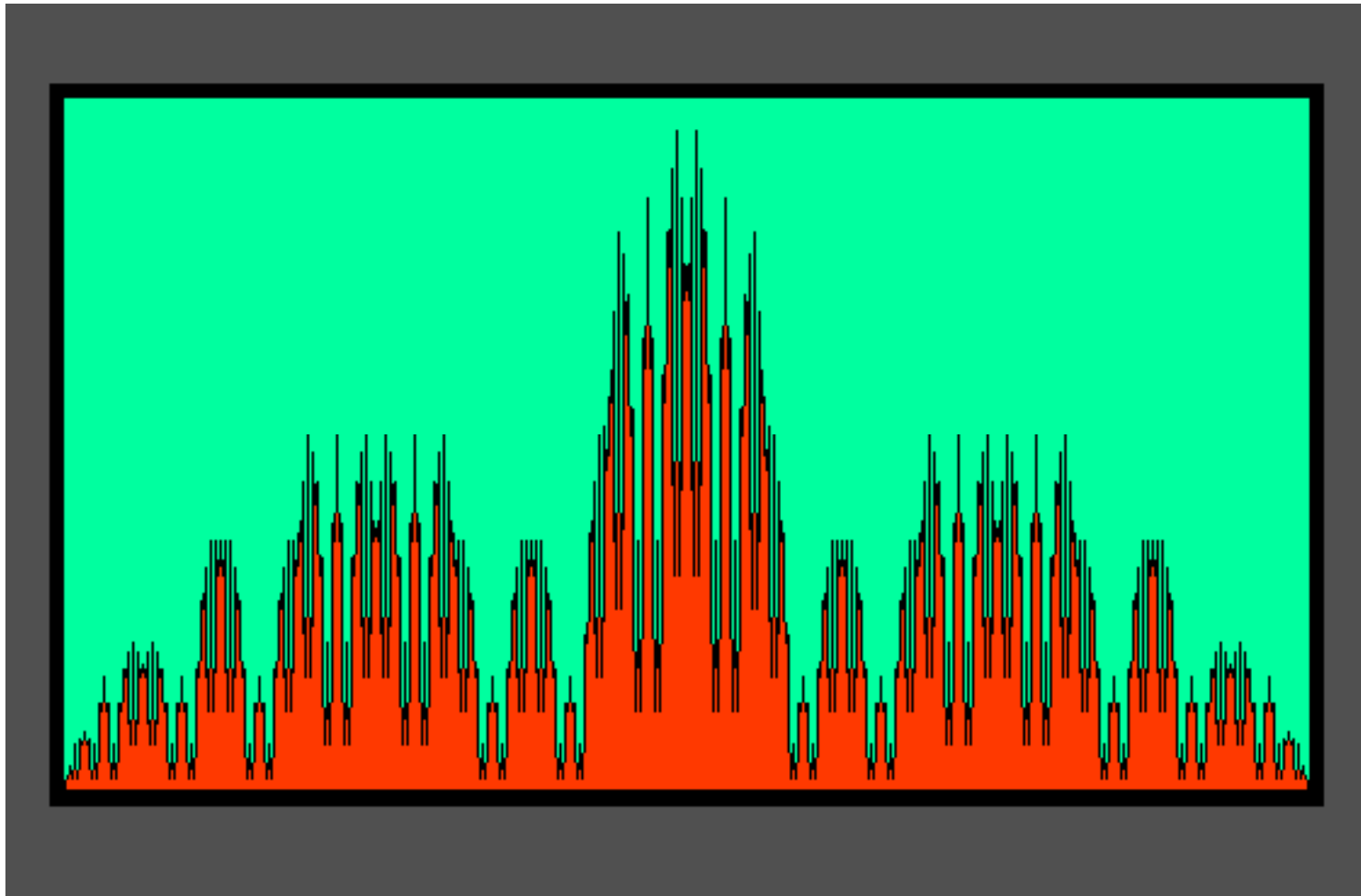
« Courbe » de Peano



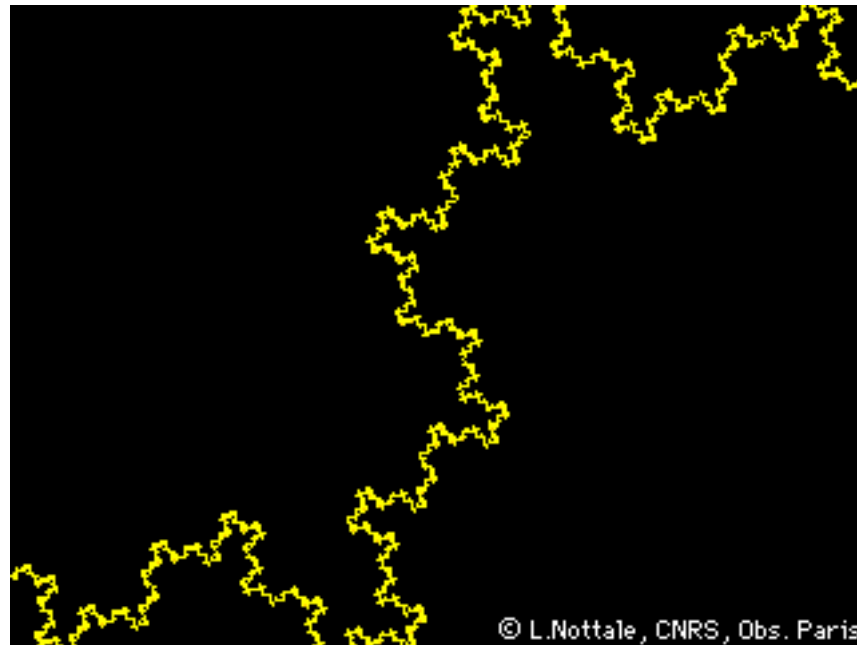
Courbe fractale $D=3/2$



Fonction fractale $D=3/2$ (projection)

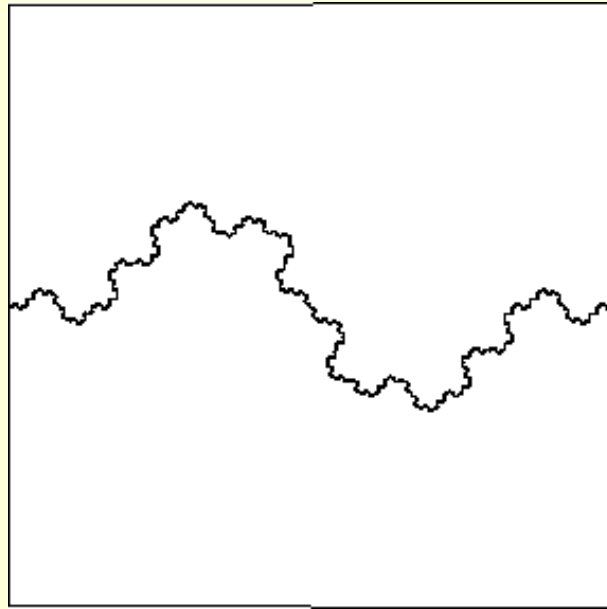


Zoom continu sur une courbe fractale

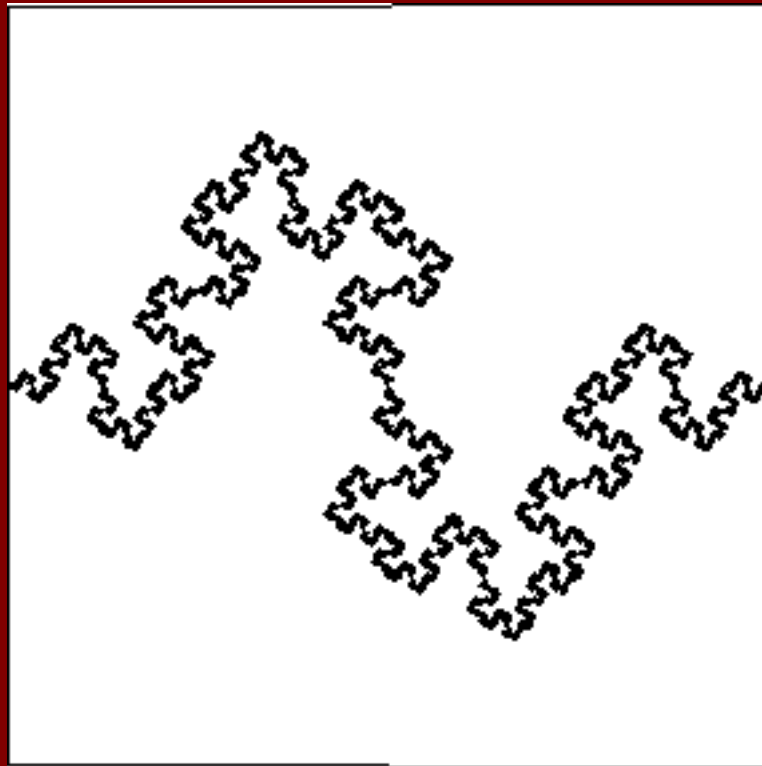


Animation

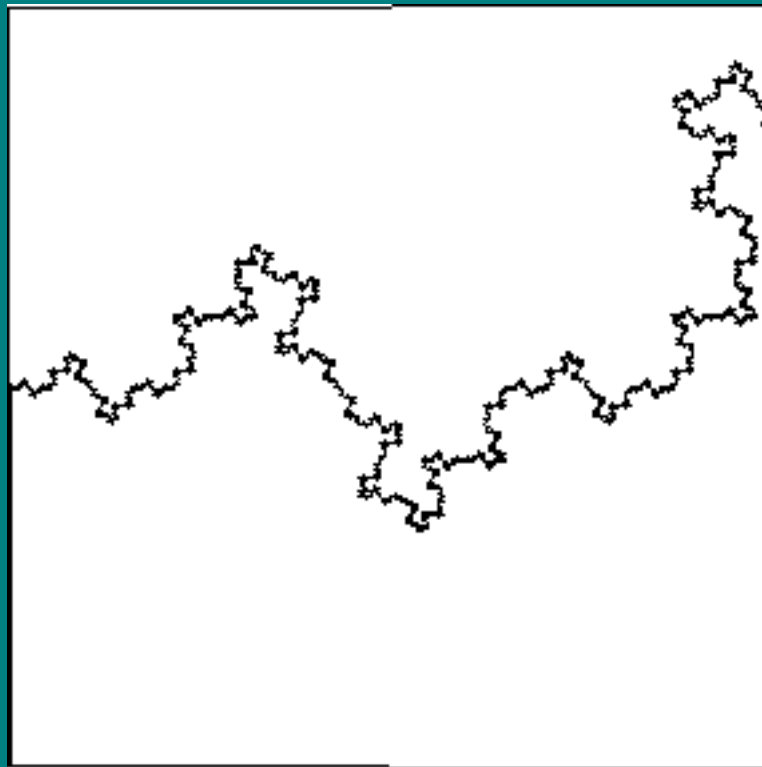
Zoom sur « zig-zag » fractal ($\pi/3$)



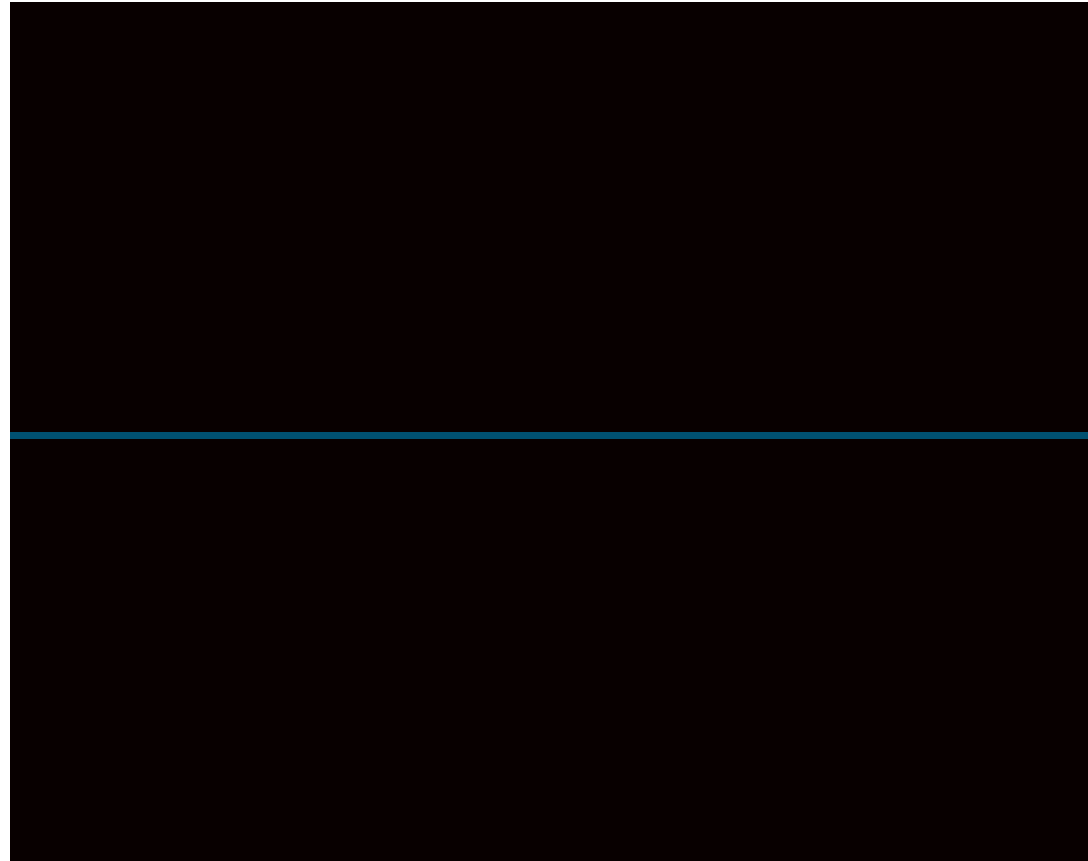
Zoom sur « zig-zag » fractal ($\pi/2$)



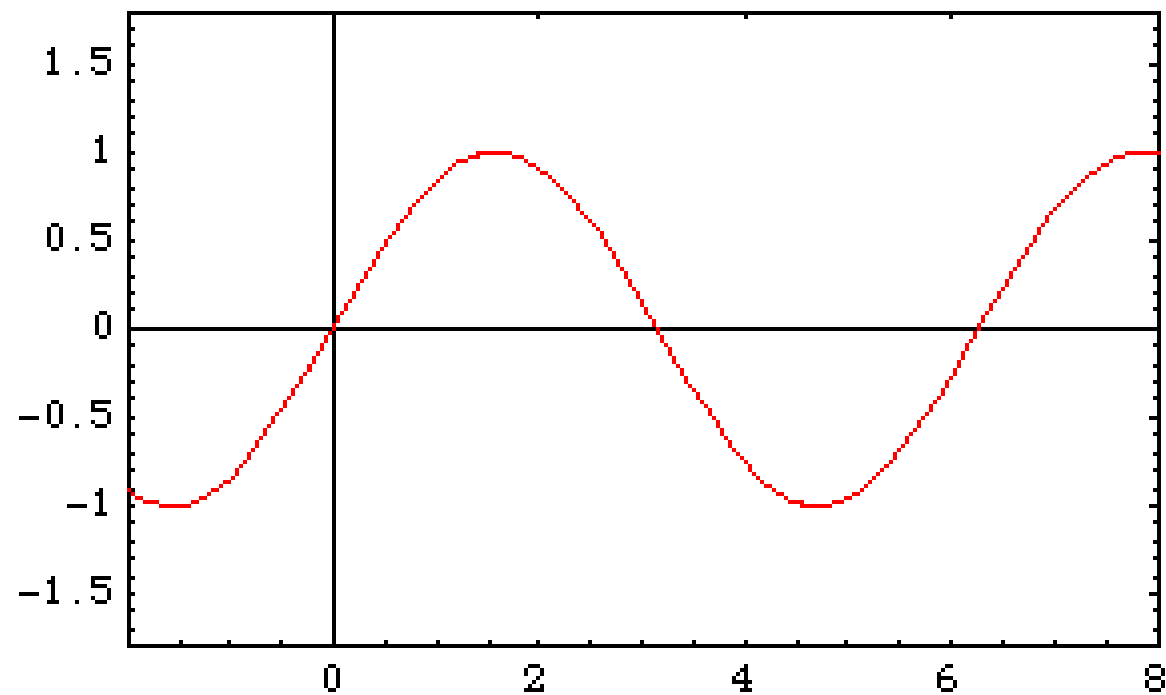
Zoom à l'origine d'une courbe fractale



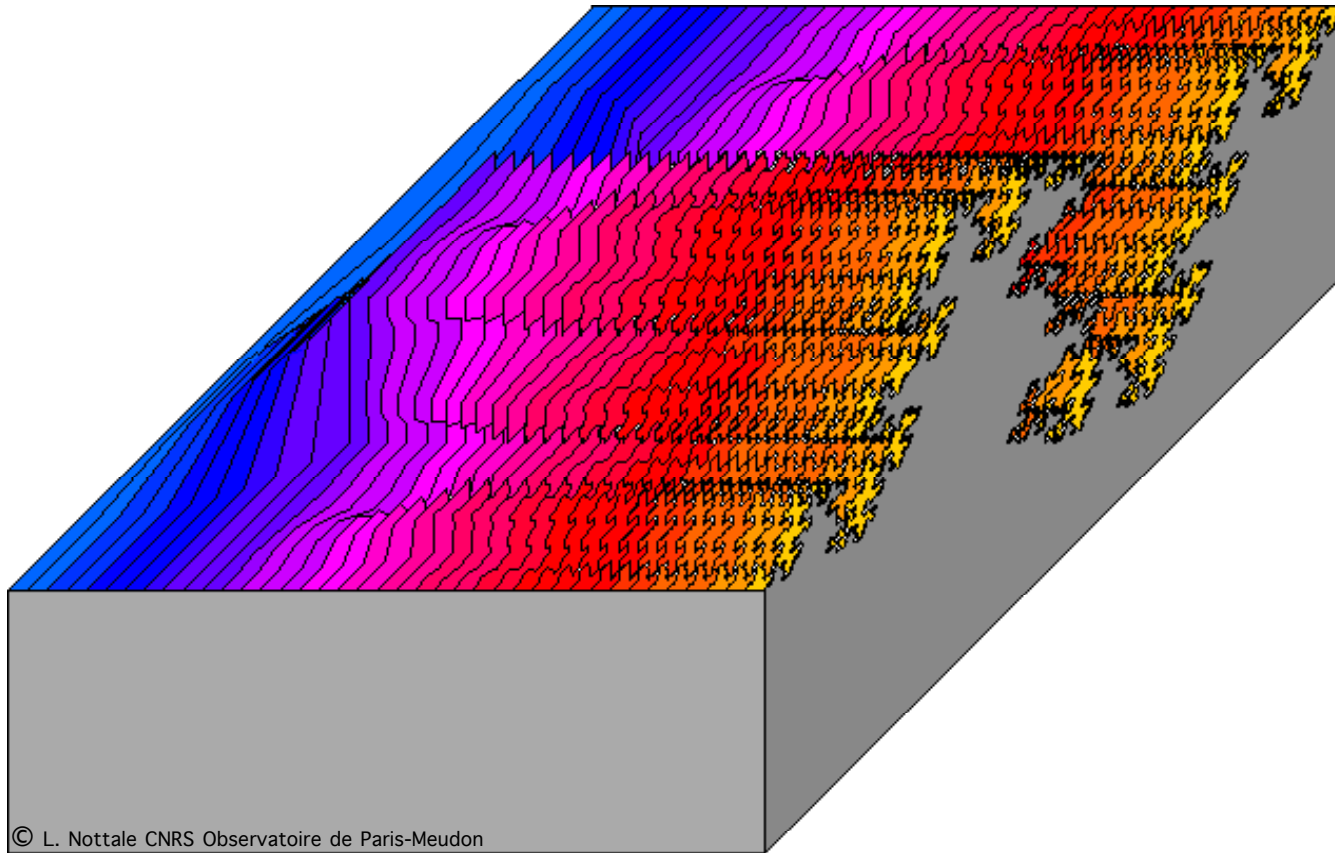
Processus de « Fractalisation »



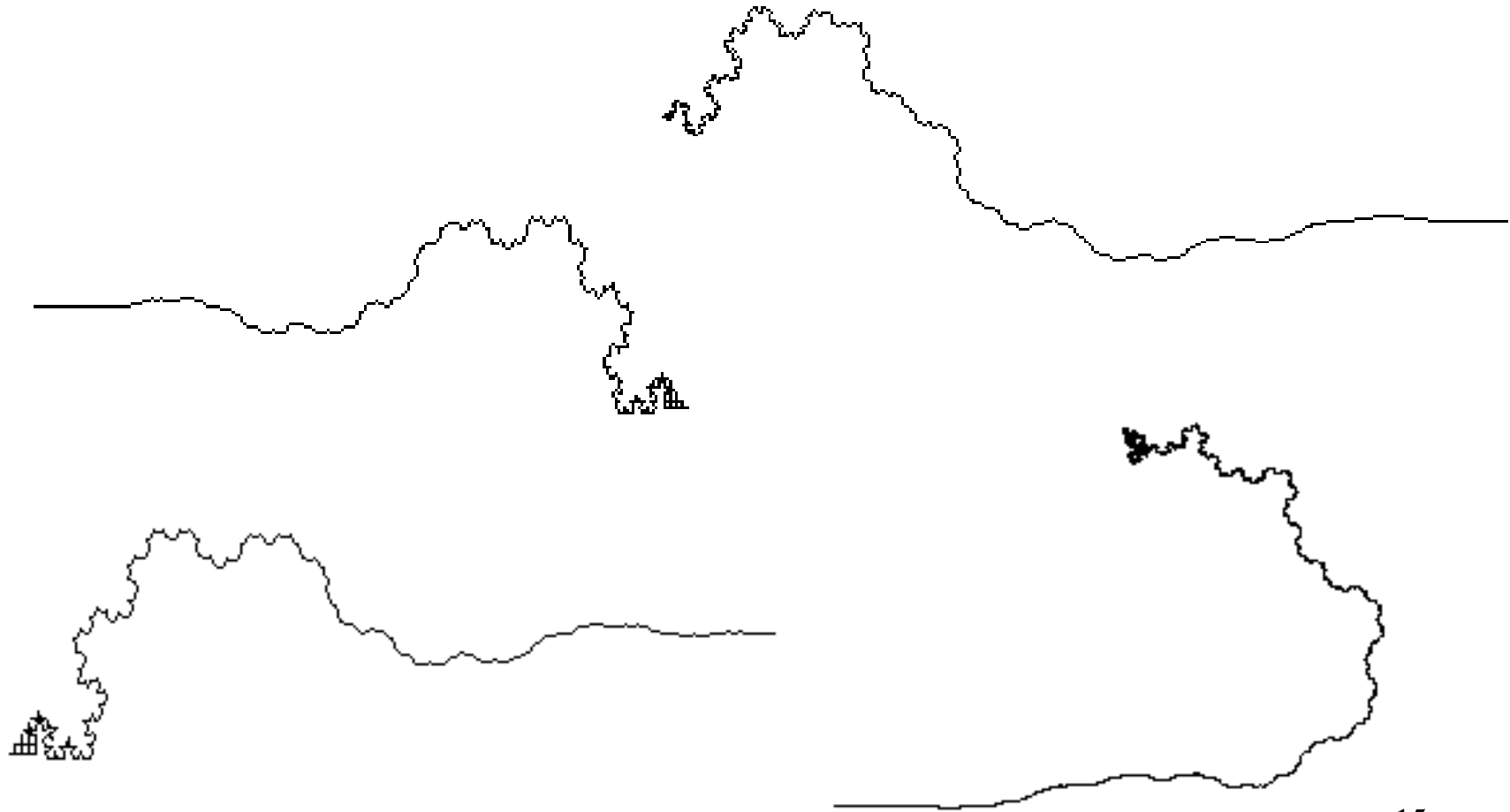
Fractalisation (courbe de Weierstrass)



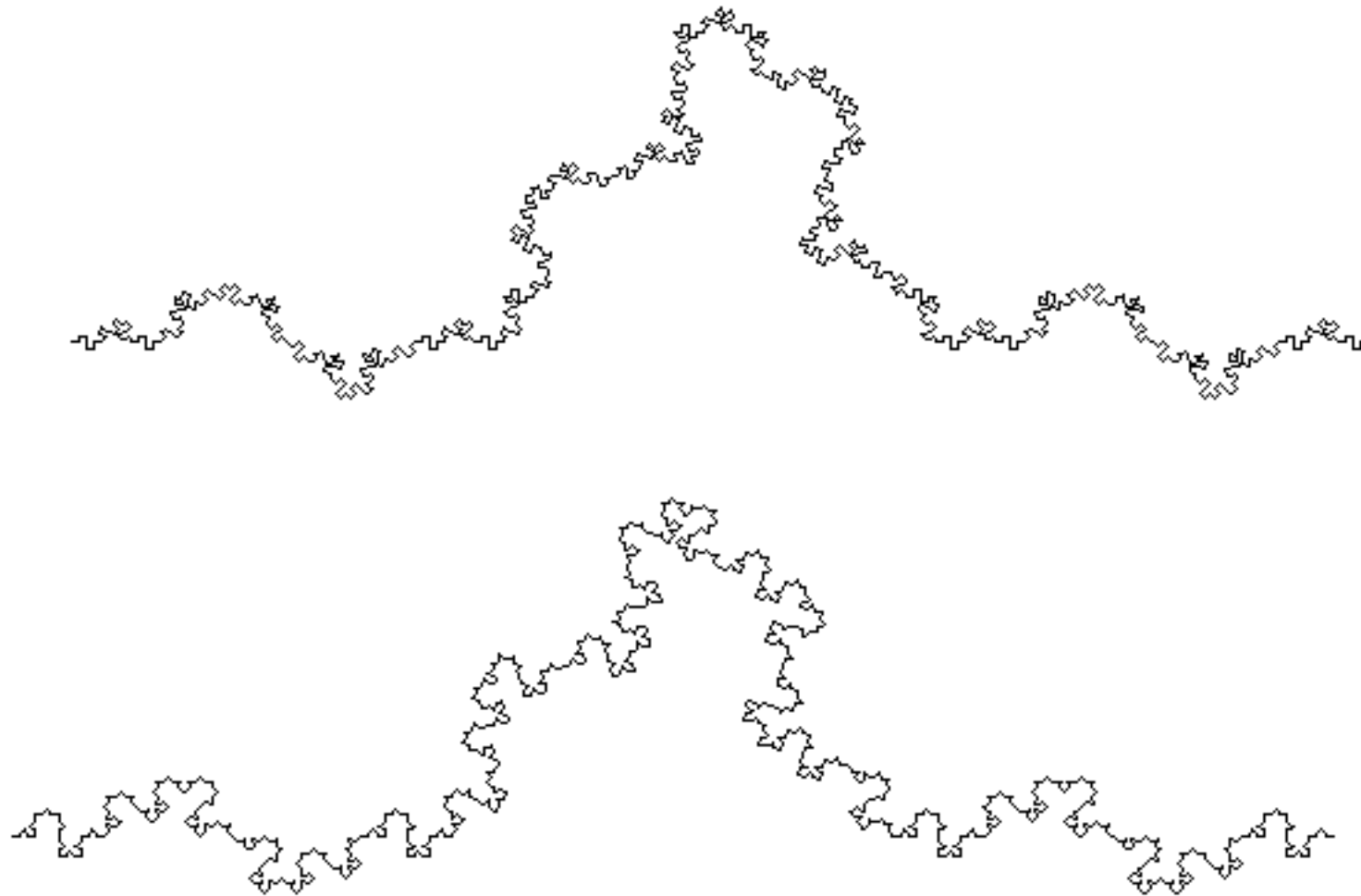
Géométrie fractale: espace des positions et des échelles



Courbes de dimension fractale variable (dans l'espace)

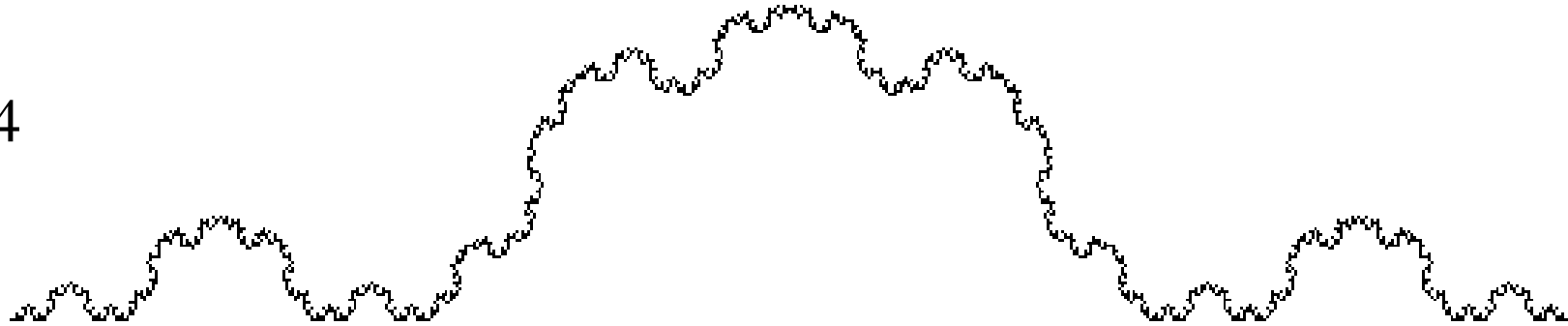


Courbes de dimension variable (en échelle):
générateur dépendant du niveau, p et q variables



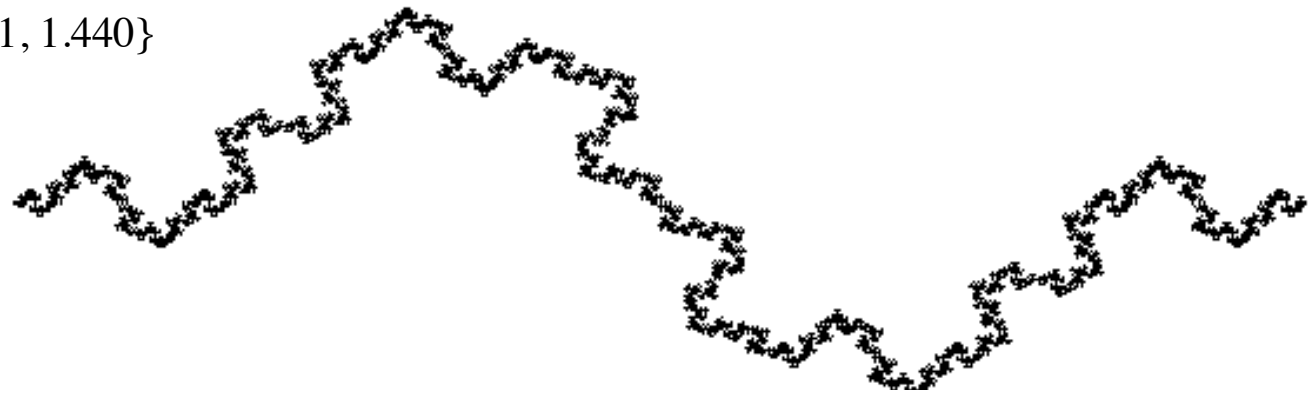
Courbes de dimension variable en échelle: générateur dépendant du niveau, p constant, q variable

$p = 4$

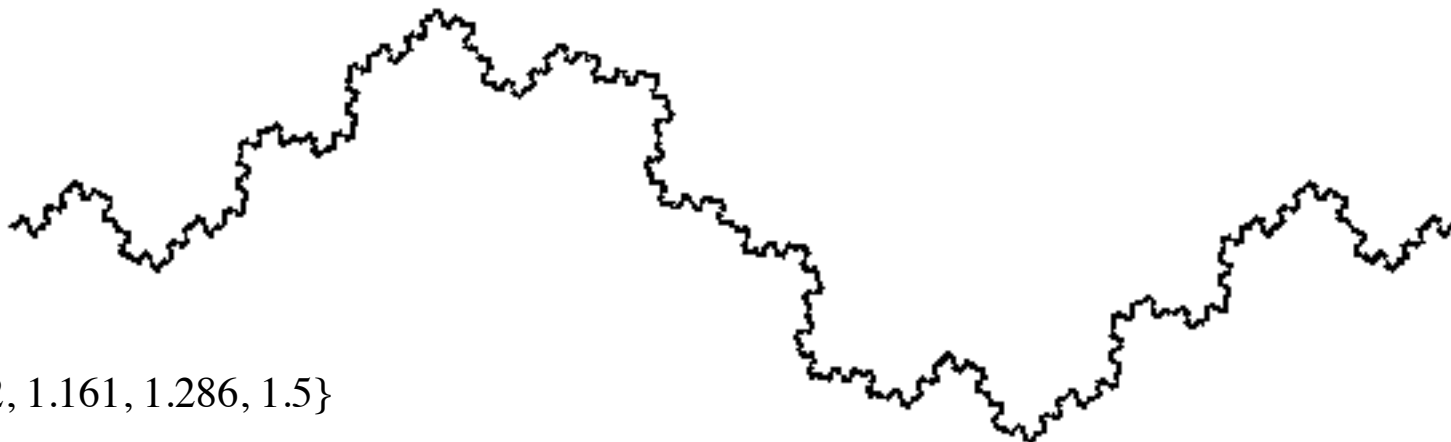


$D_F = \{1.0526, 1.078, 1.128, 1.261, 1.440\}$

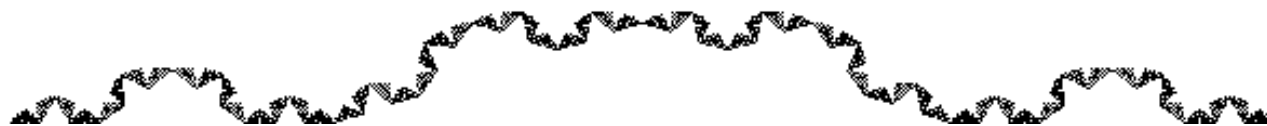
$p = 8$



$D_F = \{1.082, 1.161, 1.286, 1.5\}$



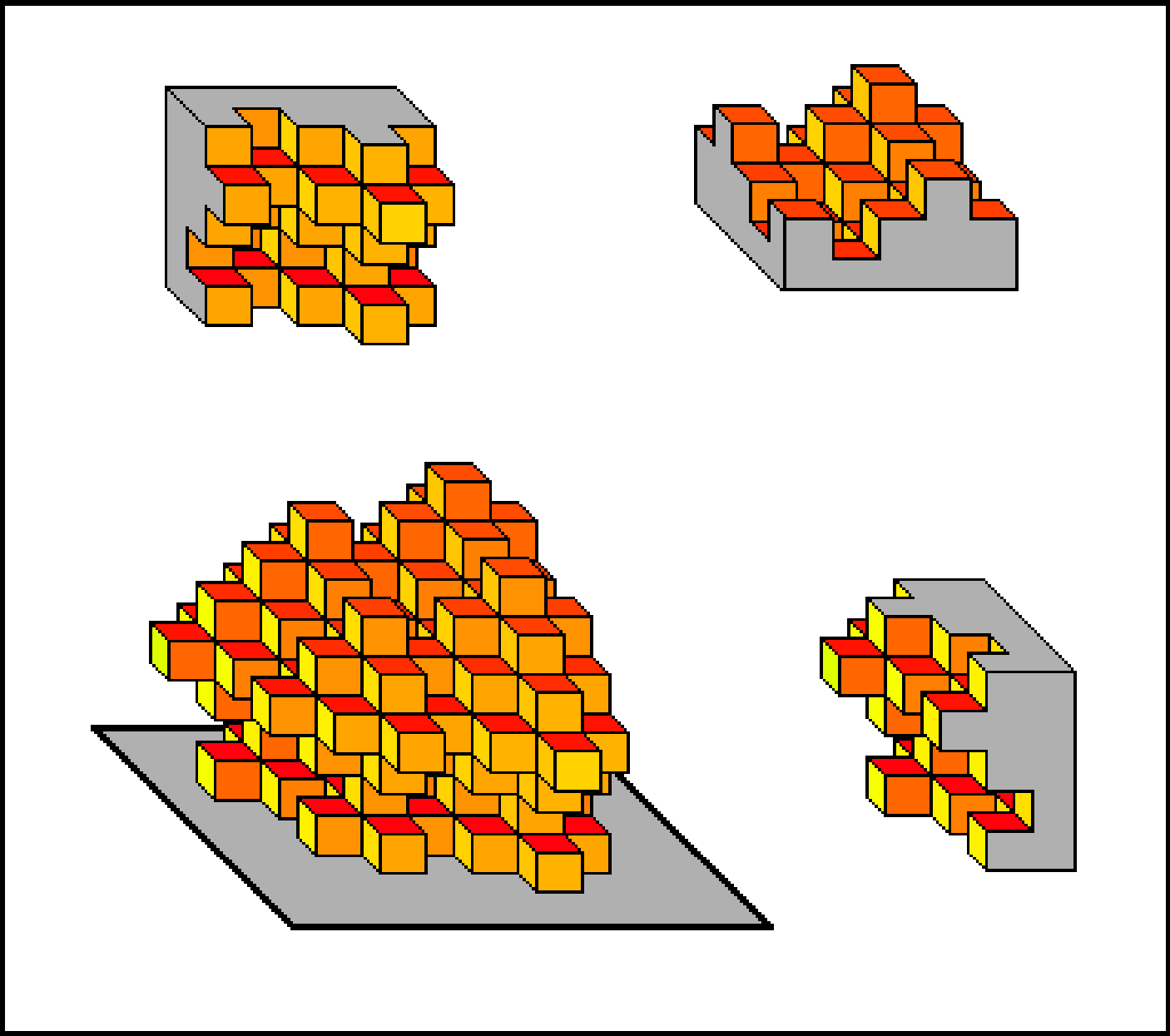
Dimension variable en échelle: zoom

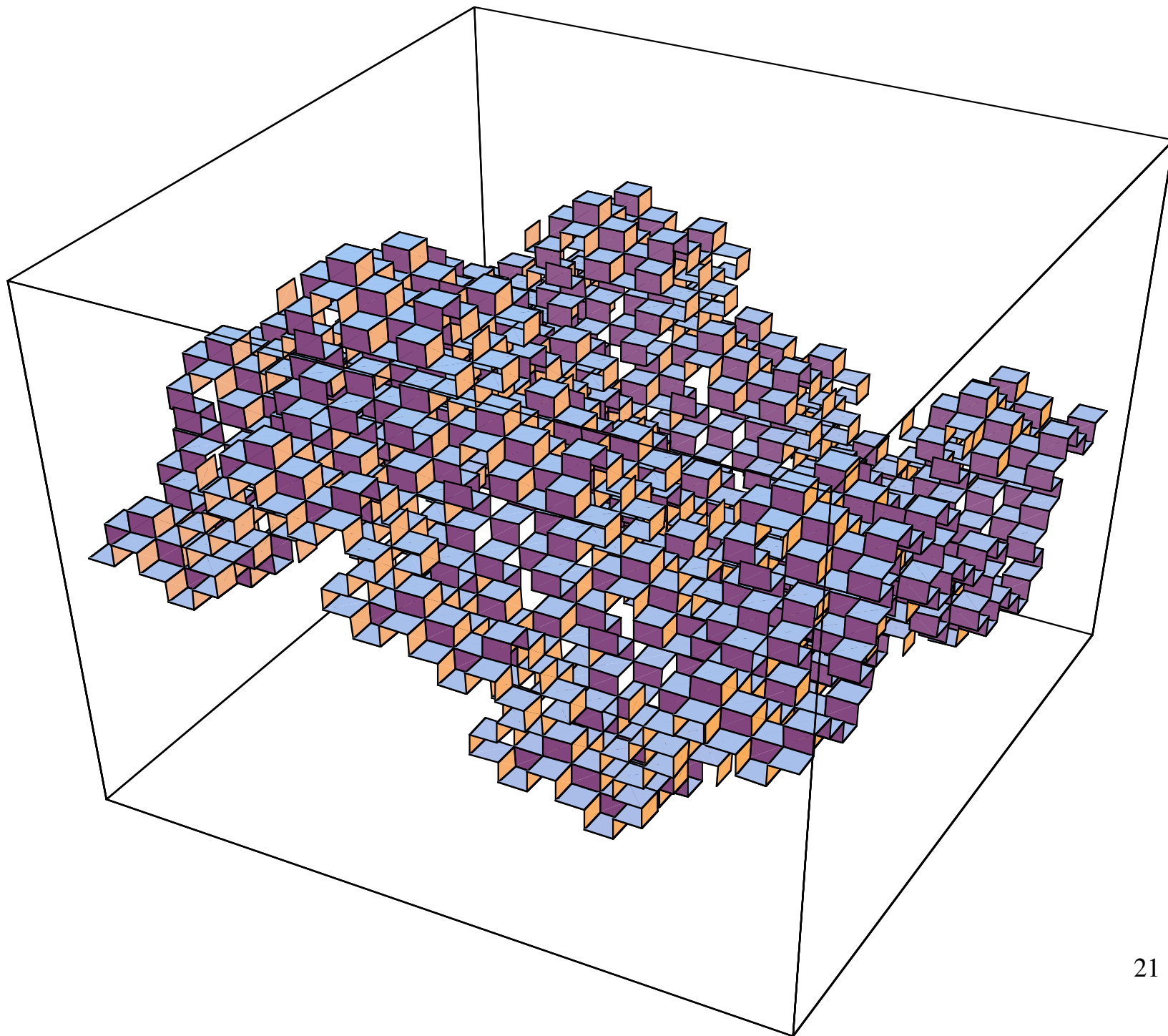


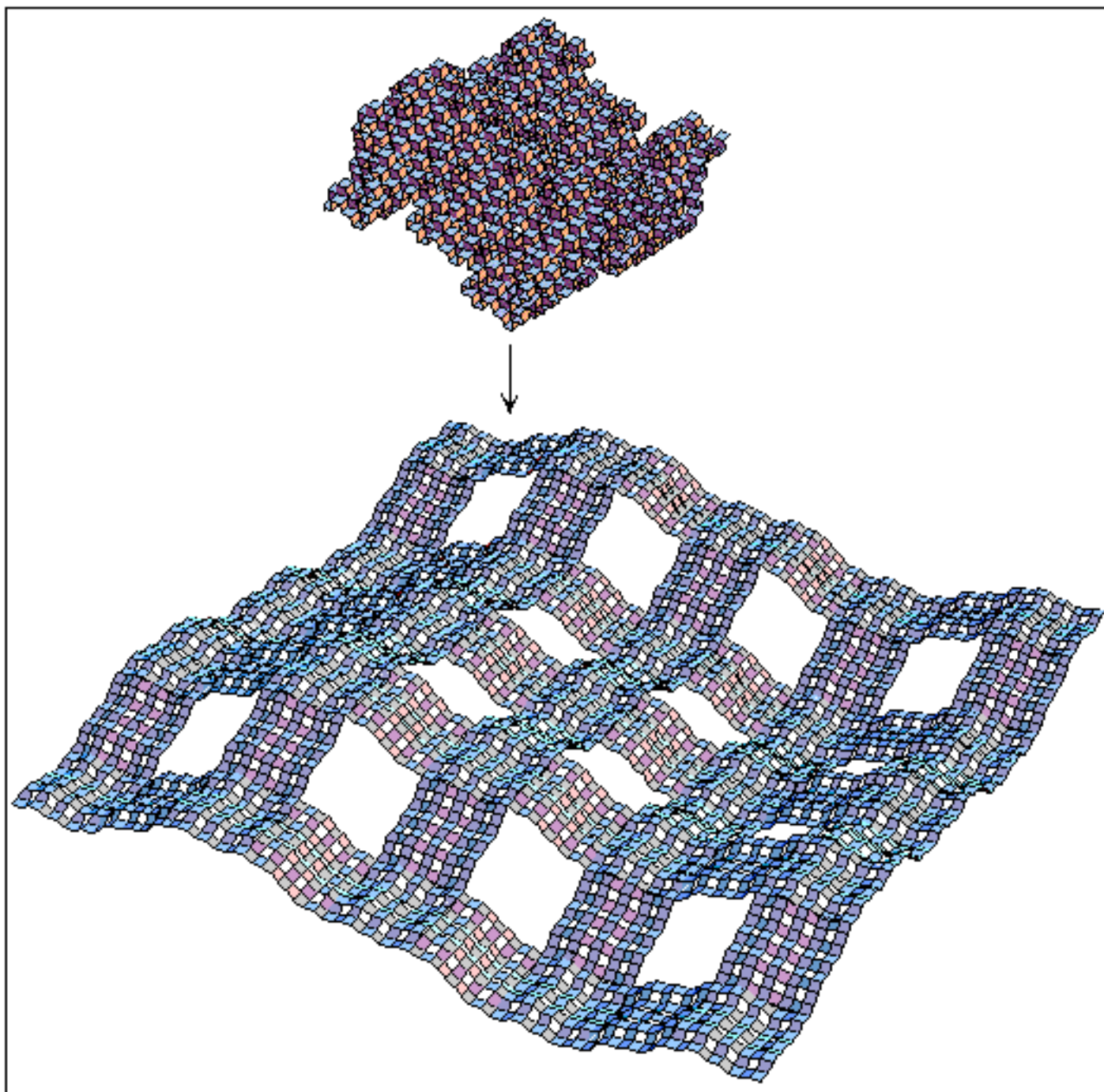
Animations

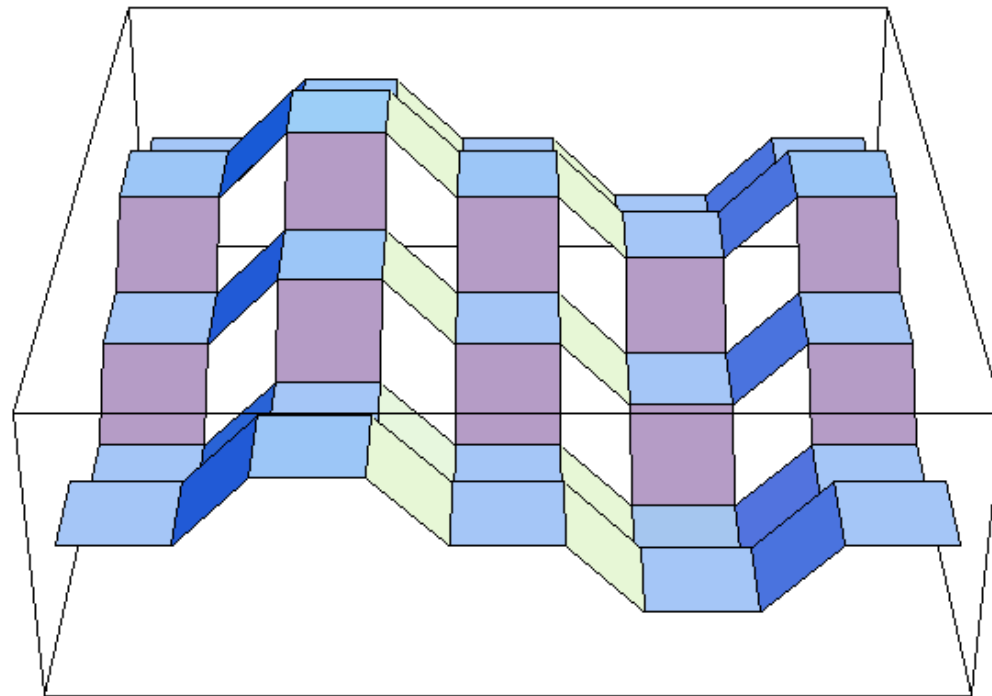
Dimension variable en échelle: zoom



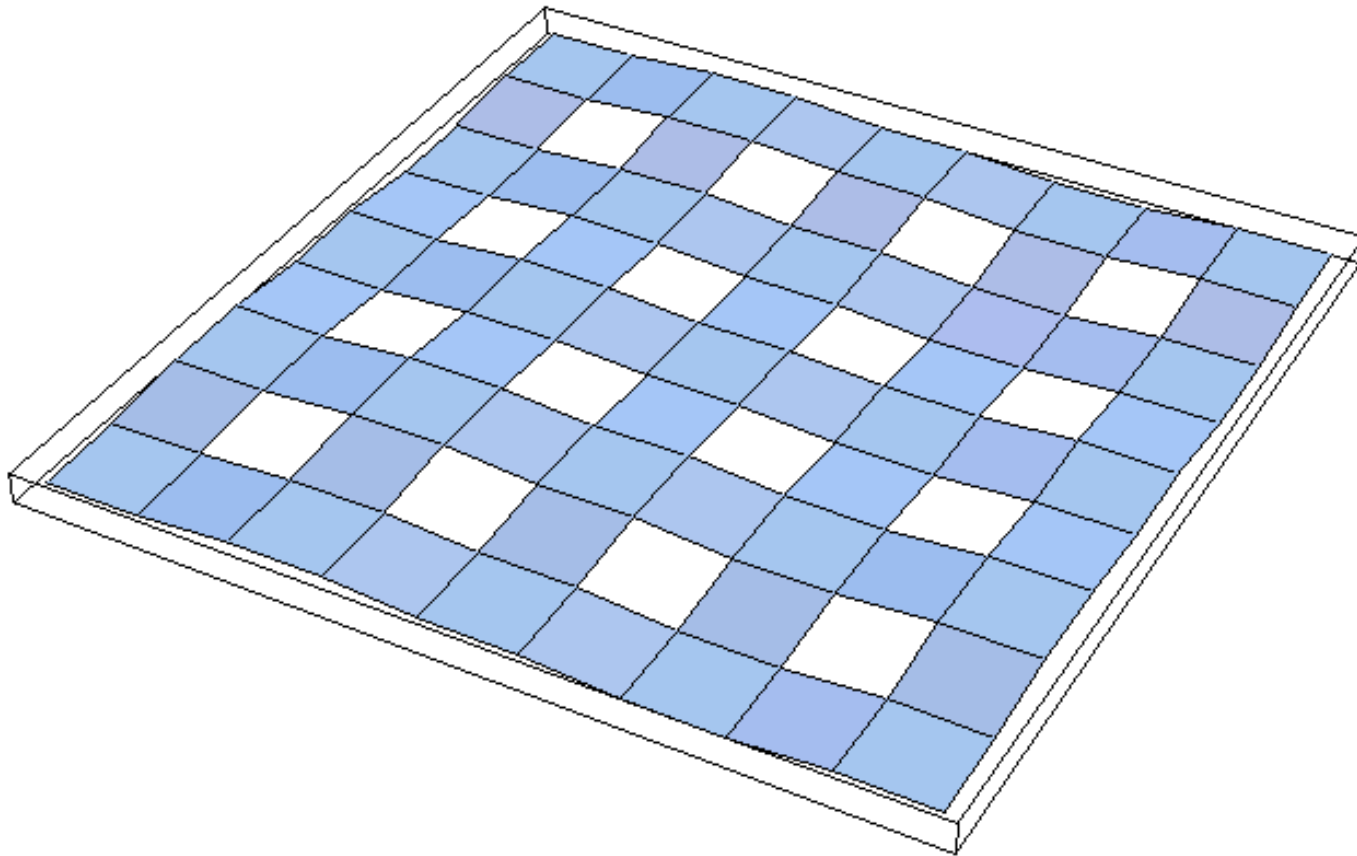




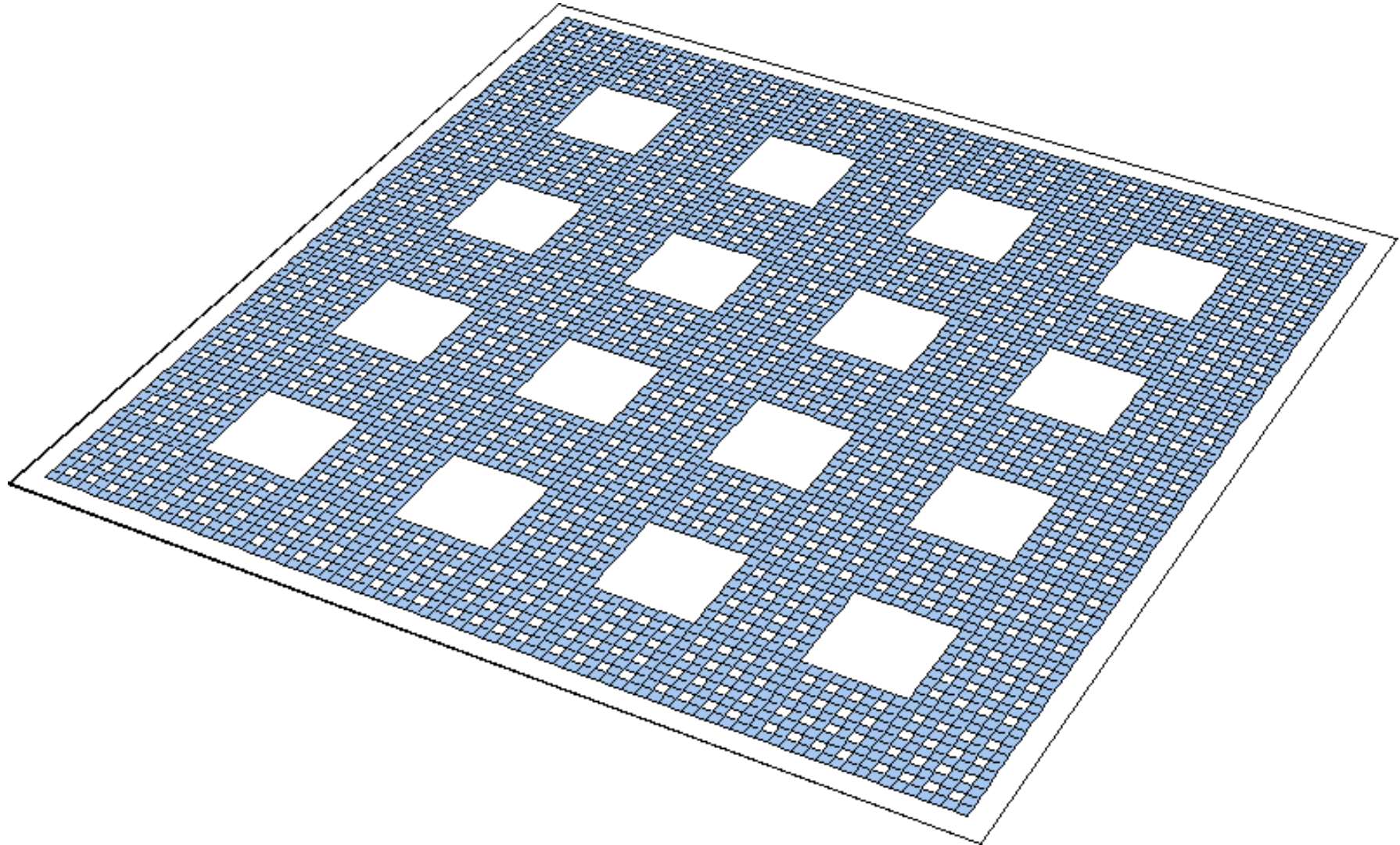




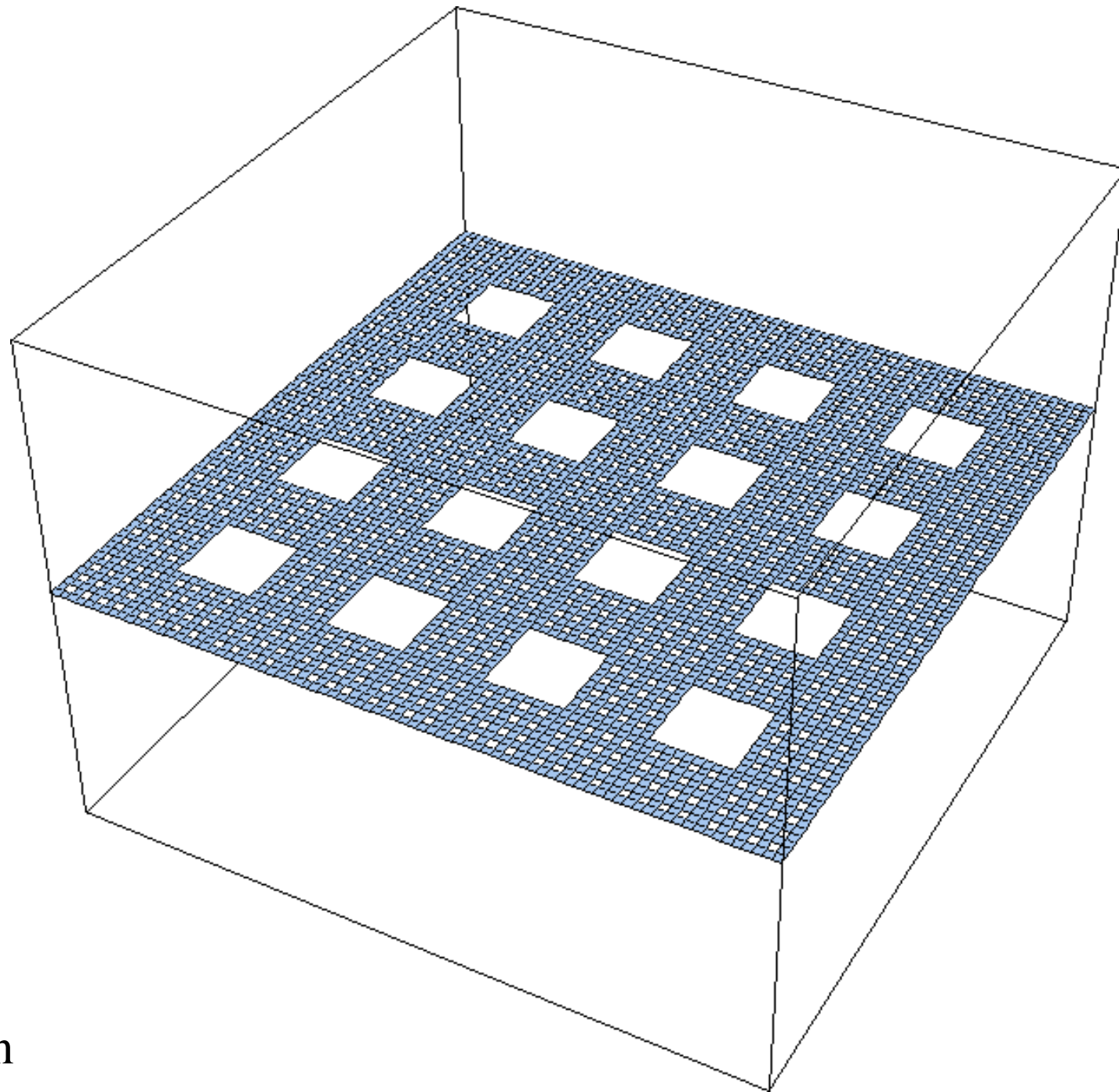
Surface fractale: repliement générateur



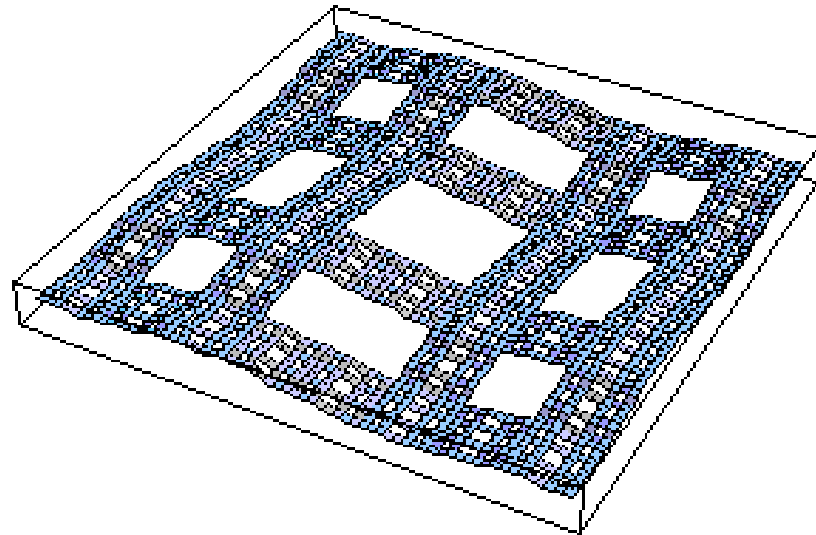
Repliement d'une surface fractale

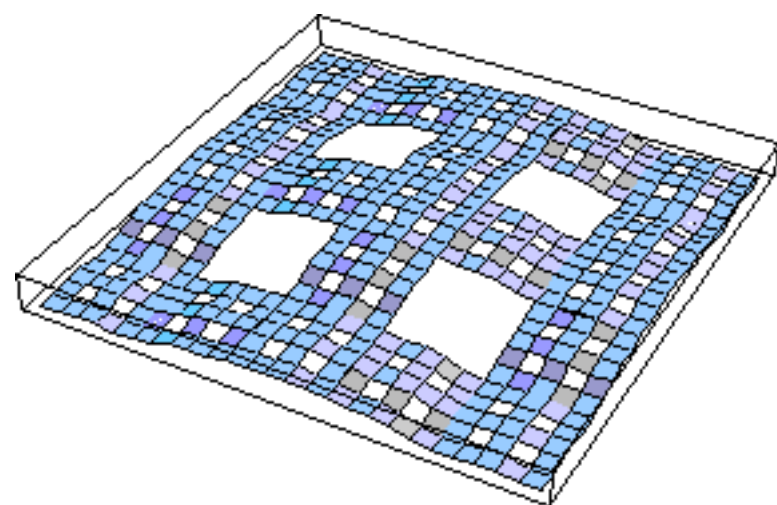


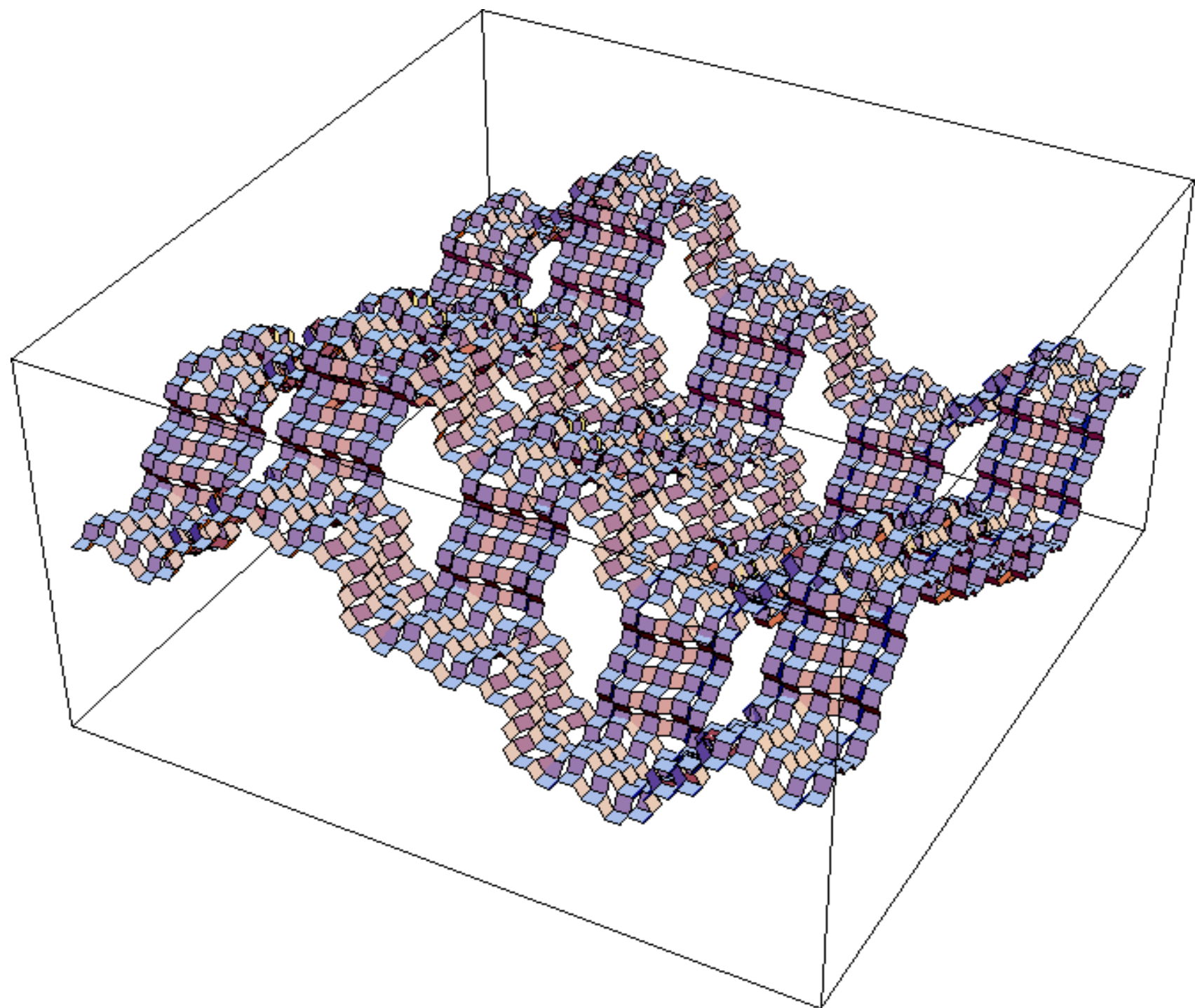
Animation



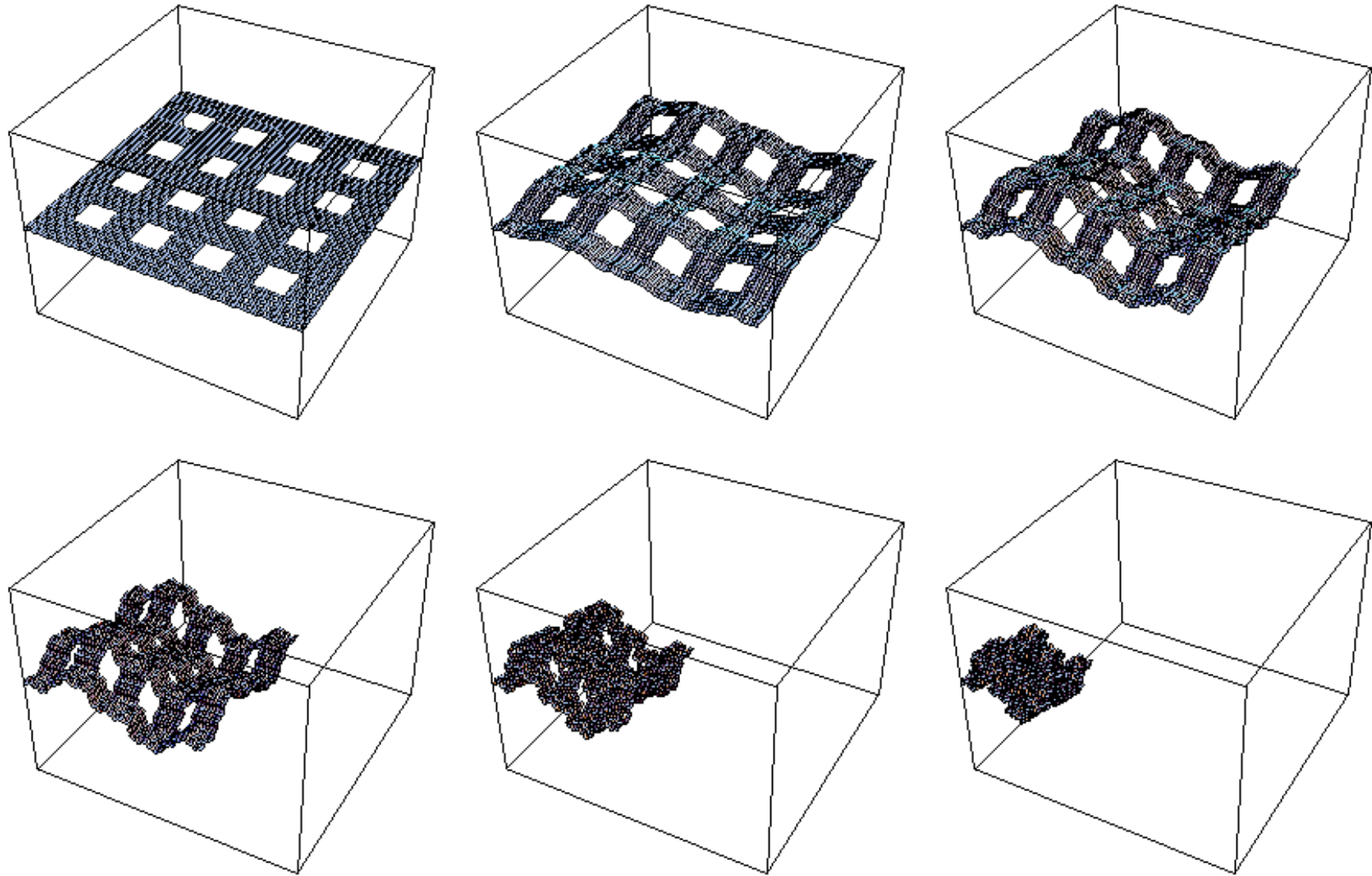
Animation



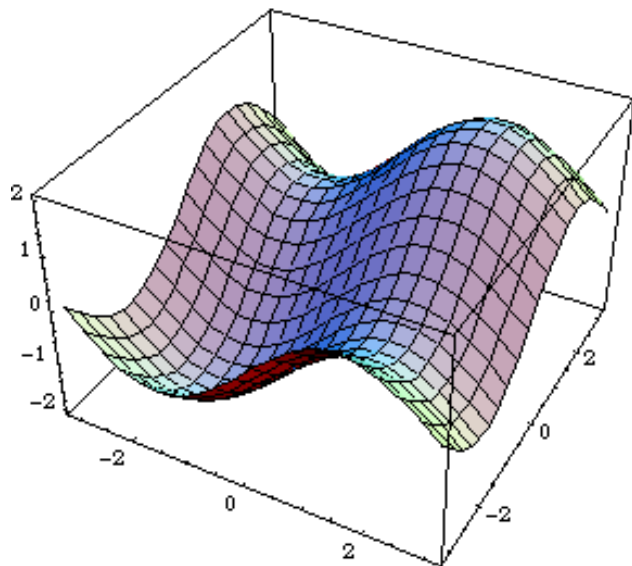




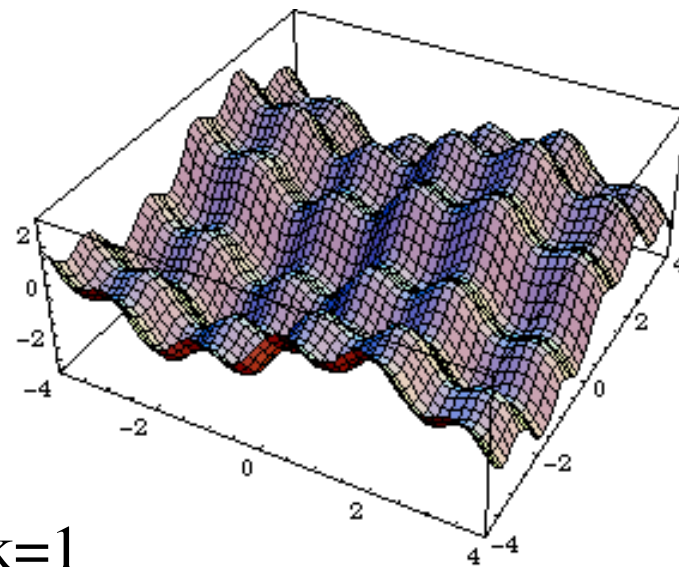
Dépliage surface fractale : étapes



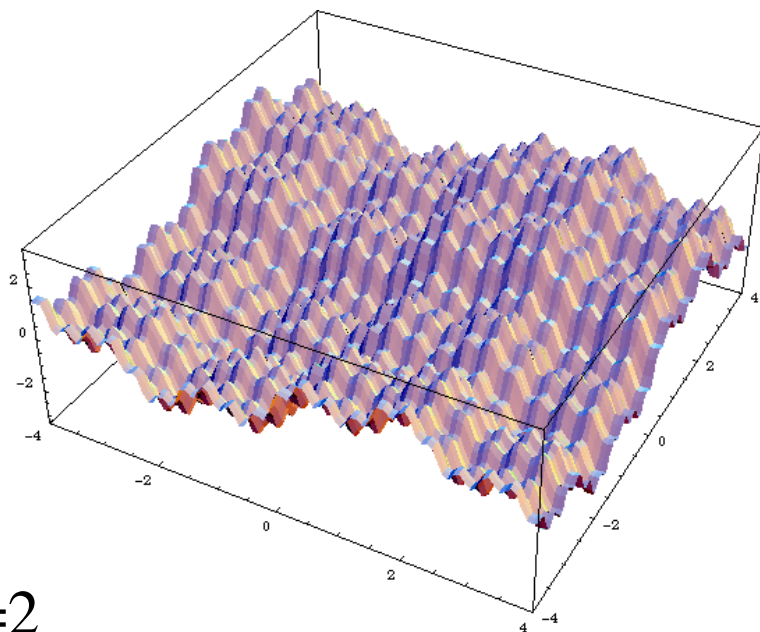
Surface fractale: dépendance d'échelle



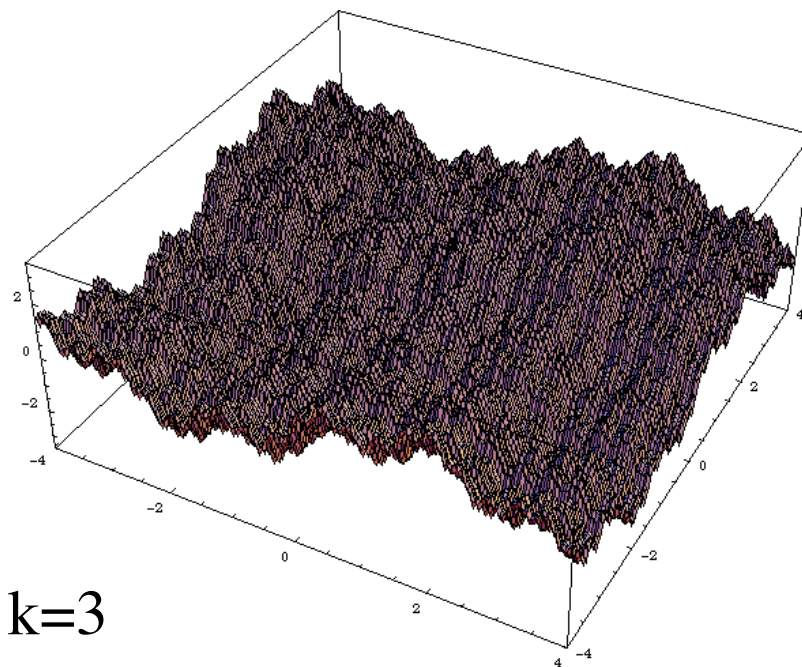
$k=0$



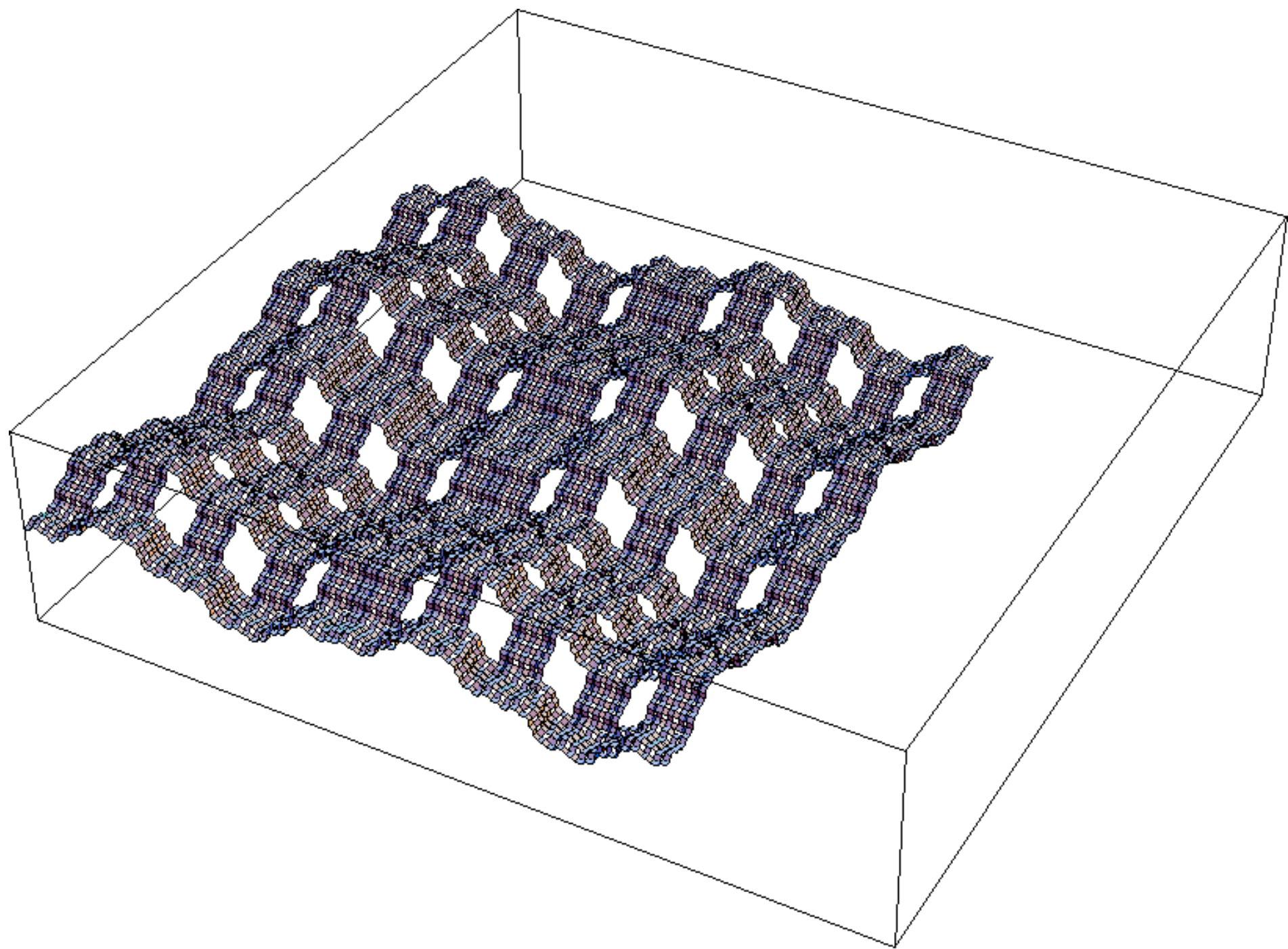
$k=1$



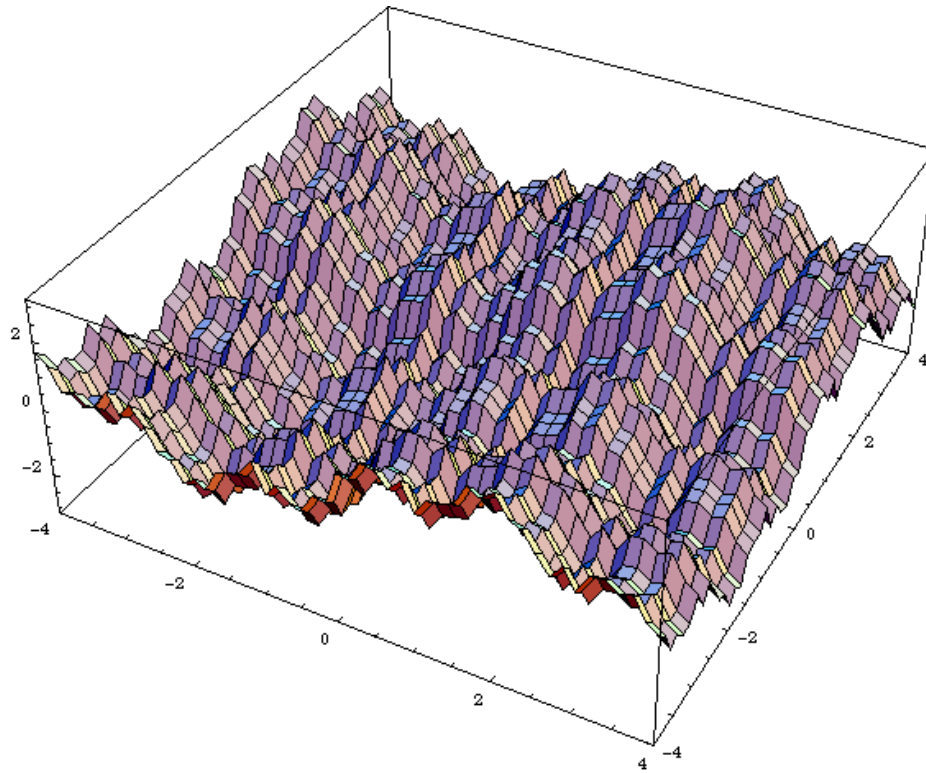
$k=2$



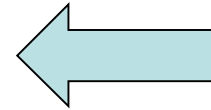
$k=3$



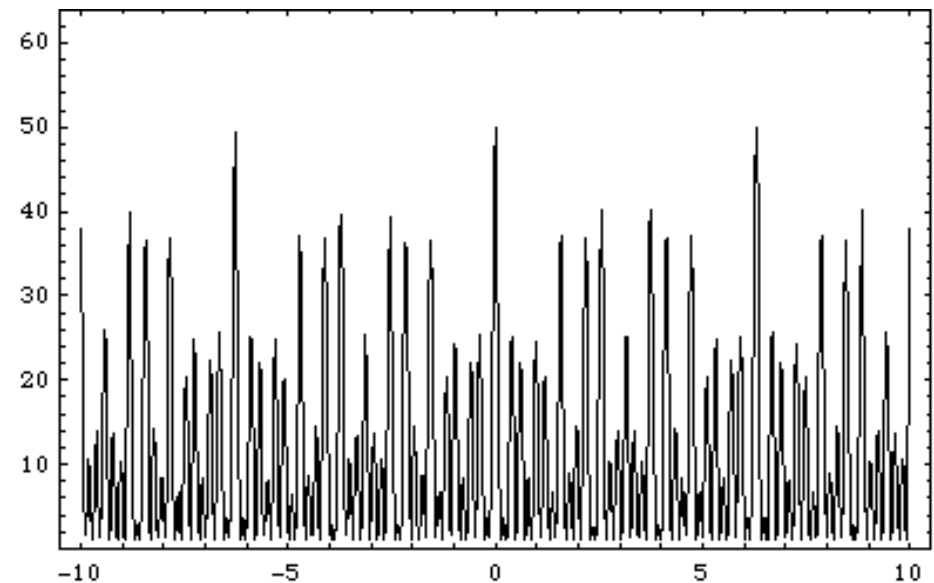
Métrie et courbure d'une surface fractale



Surface (espace
fractal de dimension
topologique 2)



Métrie
(divergente)



Courbure gaussienne d'une surface fractale

