Expérience de Stern et Gerlach et décohérence: l'ordinateur quantique est-il vraiment possible?

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Cnam 7 Mai 2012

1985 David Deutsch : Feasibility of parallel quantum computer.

1994 & 1996 Peter W. Shor, L. Glover : Quantum polynomial-time algorithms.

2001 Isaac L. Chuang & al : A 7 qubit computer using the NMR technique and Shor's algorithm to factor the number 15.

2002 Chuang abandons the NMR technique.

> Main difficulty : **decoherence destroys superposition states.**

> Objective : **understanding of decoherence and existence of spin-based qubit.**

1922 Stern-Gerlach experiment : quantization of the spin through spatial quantization.

How to explain quantization of the spin ?

- Either by the measurement postulates of quantum theory
- Either by Pauli equation with spatial extension of the spinor

Representation of the particle with spin

- Spinor complete with **spatial extension**

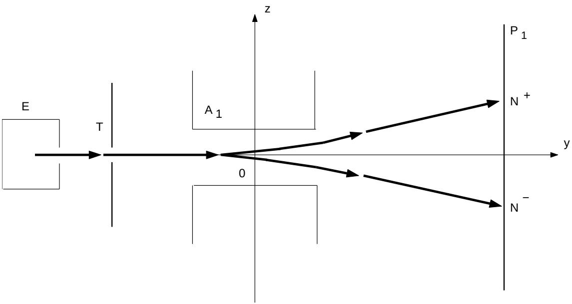
$$\Psi^0(z) = (2\pi\sigma_0^2)^{-\frac{1}{4}} e^{-\frac{z^2}{4\sigma_0^2}} \begin{pmatrix} \cos\frac{\theta_0}{2} e^{-i\frac{\varphi_0}{2}} \\ \sin\frac{\theta_0}{2} e^{i\frac{\varphi_0}{2}} \end{pmatrix} \quad (1)$$

- Simplified spinor used in quantum information (qubit)

$$\Psi^0 = \begin{pmatrix} \cos\frac{\theta_0}{2} e^{-i\frac{\varphi_0}{2}} \\ \sin\frac{\theta_0}{2} e^{i\frac{\varphi_0}{2}} \end{pmatrix}. \quad (2)$$

Outline

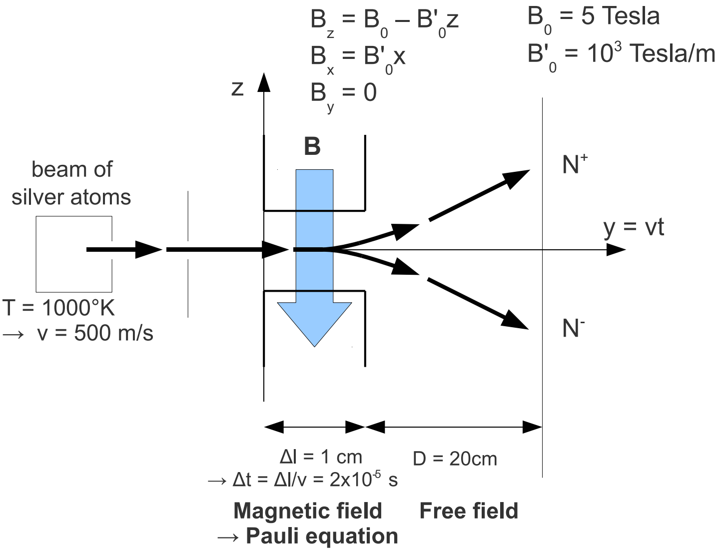
1. Stern-Gerlach experiment
2. Decoherence in Stern-Gerlach experiment
3. Demonstration postulats quantification, décomposition spatiale
4. Impacts and quantization explained by trajectories
5. Conclusion on quantum computer



$$\Psi^0(z) = (2\pi\sigma_0^2)^{-\frac{1}{4}} e^{-\frac{z^2}{4\sigma_0^2}} \begin{pmatrix} \cos\frac{\theta_0}{2} e^{-i\frac{\varphi_0}{2}} \\ \sin\frac{\theta_0}{2} e^{i\frac{\varphi_0}{2}} \end{pmatrix} \tag{3}$$

- **pure state** : θ_0 and φ_0 fixed
- **mixed states** : θ_0 and φ_0 randomly drawn

$\sigma_0 = 10^{-4}$m


$B_z = B_0 - B'_0 z$
$B_x = B'_0 x$
$B_y = 0$
$B_0 = 5$ Tesla
$B'_0 = 10^3$ Tesla/m
z
B
beam of silver atoms
N^+
y = vt
N^-
T = 1000°K
→ v = 500 m/s
Δl = 1 cm
→ Δt = Δl/v = 2x10^{-5} s
D = 20cm
Magnetic field
→ Pauli equation
Free field

Stern-Gerlach experiment

- Pauli equation for spinor $\Psi = \binom{\psi_+}{\psi_-}$ in magnetic field $\mathbf{B}$:

$$i\hbar \begin{pmatrix} \frac{\partial \psi_+}{\partial t} \\ \frac{\partial \psi_-}{\partial t} \end{pmatrix} = -\frac{\hbar^2}{2m}\Delta \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} + \mu_B \mathbf{B}\boldsymbol{\sigma} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \qquad (4)$$

- $\mu_B = \frac{e\hbar}{2m_e}$: Bohr magneton
- $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$: Pauli matrices

- Pauli equation for spinor $\Psi = \binom{\psi_+}{\psi_-}$ in free space :

$$i\hbar \begin{pmatrix} \frac{\partial \psi_+}{\partial t} \\ \frac{\partial \psi_-}{\partial t} \end{pmatrix} = -\frac{\hbar^2}{2m}\Delta \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \qquad (5)$$

Decoherence in Stern-Gerlach experiment

Time in the magnetic field

$$\Delta t = \frac{\Delta l}{v} = 2 \times 10^{-5} s$$

After the magnetic field : at $t + \Delta t$

$$\Psi(z, t+\Delta t) \simeq (2\pi\sigma_0^2)^{-\frac{1}{4}} \begin{pmatrix} \cos\frac{\theta_0}{2} e^{-\frac{(z-z_\Delta-ut)^2}{4\sigma_0^2}} e^{i\frac{muz+\hbar\varphi_+}{\hbar}} \\ \sin\frac{\theta_0}{2} e^{-\frac{(z+z_\Delta+ut)^2}{4\sigma_0^2}} e^{i\frac{-muz+\hbar\varphi_-}{\hbar}} \end{pmatrix} \quad (6)$$

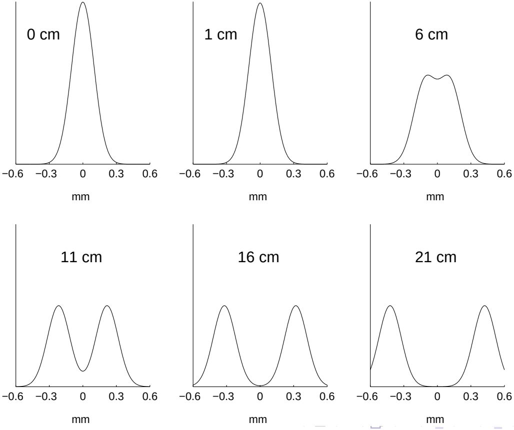
with

$$z_\Delta = \frac{\mu_B B_0'(\Delta t)^2}{2m} = 10^{-5} m, \qquad u = \frac{\mu_B B_0'(\Delta t)}{m} = 1 m/s. \quad (7)$$

Decoherence in Stern-Gerlach experiment

$$\rho(z, t+\Delta t) \quad \simeq (2\pi\sigma_0^2)^{-\frac{1}{2}} \frac{1}{2} \left(e^{-\frac{(z-z_\Delta-ut)^2}{2\sigma_0^2}} + e^{-\frac{(z+z_\Delta+ut)^2}{2\sigma_0^2}} \right) \quad (8)$$

$y = vt$



Decoherence in Stern-Gerlach experiment

The decoherence time

Spots N^+ and N^- appear : $y = vt >$16 cm

$\Rightarrow$ **the decoherence time :**

$$t_D \simeq \frac{3\sigma_0 - z_\Delta}{u} = \frac{(3\sigma_0 - z_\Delta)mv}{\mu_B B_0' \Delta l} = 3 \times 10^{-4} s. \qquad (9)$$

Marginal density matrix of spin variables of a pure state

$$\rho^S(z,t) = \begin{pmatrix} |\psi_+(z,t)|^2 & \psi_+(z,t)\psi_-^*(z,t) \\ \psi_-(z,t)\psi_+^*(z,t) & |\psi_-(z,t)|^2 \end{pmatrix} \qquad (10)$$

When $t > t_D$:

$$\rho^S(z,t) \simeq \begin{pmatrix} |\psi_+(z,t)|^2 & 0 \\ 0 & |\psi_-(z,t)|^2 \end{pmatrix} \qquad (11)$$

Decoherence in Stern-Gerlach experiment

How is the transformation done ?

mixed states

θ_0 and φ_0 randomly drawn

$\Longrightarrow$

quantized mixture

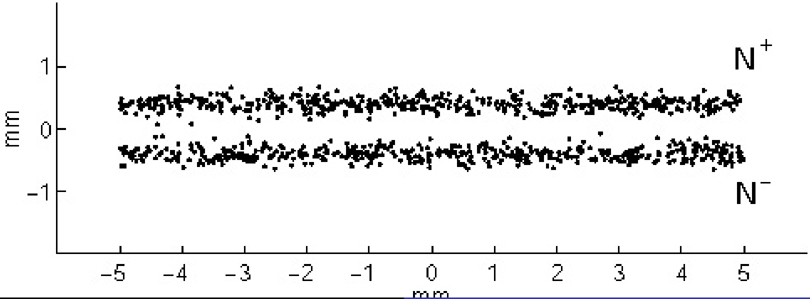
$\theta_0 = \pi$ and $\theta_0 = 0$

quantization postulate of the measure
or
Pauli equation with spatial extension of the spinor

Proof of the quantization postulate for $S_z = \frac{\hbar}{2}\sigma_z$

$$\Psi(z,t+\Delta t) \simeq (2\pi\sigma_0^2)^{-\frac{1}{4}} \begin{pmatrix} \cos\frac{\theta_0}{2} e^{-\frac{(z-z_\Delta-ut)^2}{4\sigma_0^2}} e^{i\frac{muz+\hbar\varphi_+}{\hbar}} \\ \sin\frac{\theta_0}{2} e^{-\frac{(z+z_\Delta+ut)^2}{4\sigma_0^2}} e^{i\frac{-muz+\hbar\varphi_-}{\hbar}} \end{pmatrix} \quad (12)$$

Experimentally, we measure the particle position $\tilde{z}$



Demonstration postulats quantification, décomposition spatiale

Proof of the quantization postulate for $S_z = \frac{\hbar}{2}\sigma_z$

$\tilde{z}_1 \in N^+$

$$\Psi(\tilde{z}_1, t+\Delta t) \simeq (2\pi\sigma_0^2)^{-\frac{1}{4}} \cos\frac{\theta_0}{2} e^{-\frac{(\tilde{z}_1 - z_\Delta - ut)^2}{4\sigma_0^2}} e^{i\frac{mu\tilde{z}_1 + \hbar\varphi_+}{\hbar}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$\tilde{z}_2 \in N^-$

$$\Psi(\tilde{z}_2, t+\Delta t) \simeq (2\pi\sigma_0^2)^{-\frac{1}{4}} \sin\frac{\theta_0}{2} e^{-\frac{(\tilde{z}_2 + z_\Delta + ut)^2}{4\sigma_0^2}} e^{i\frac{-mu\tilde{z}_2 + \hbar\varphi_-}{\hbar}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ et $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ sont les vecteurs propres de σ_z.

Preuve du postulat de décomposition spatiale
Pour $t \geq t_D$ la densité de probabilité de l'état pur

$$\rho(z, t+\Delta t) \quad = (2\pi\sigma_0^2)^{-\frac{1}{2}} \left(\cos^2 \frac{\theta_0}{2} e^{-\frac{(z - z_\Delta - ut)^2}{2\sigma_0^2}} + \sin^2 \frac{\theta_0}{2} e^{-\frac{(z + z_\Delta + ut)^2}{2\sigma_0^2}} \right) \quad (13)$$

admet comme support les deux taches N^- et N^+.
On obtient une démonstration du **postulat de la décomposition spatiale** en intégrant l'équation (13) sur N^+(resp.N^-) ; on trouve la probabilité $\cos^2 \frac{\theta_0}{2}$ (resp.$\sin^2 \frac{\theta_0}{2}$) de mesurer la particule dans l'état de spin $+\frac{\hbar}{2}$ (resp.$-\frac{\hbar}{2}$).

Impacts and quantization explained by trajectories

Initial spatial extension of the spinor

- initial density : $\rho^0(z) = (2\pi\sigma_0^2)^{-1} e^{-\frac{z^2}{2\sigma_0^2}}$.
- Classical particles of same density have the position z_0 randomly drawn in this density

The trajectories in de Broglie-Bohm interpretation

- velocity :

$$\mathbf{v} = \frac{\hbar}{2m\rho} Im(\Psi^\dagger \boldsymbol{\nabla} \Psi) + \frac{\hbar}{2m\rho} rot(\Psi^\dagger \boldsymbol{\sigma} \Psi) \tag{14}$$

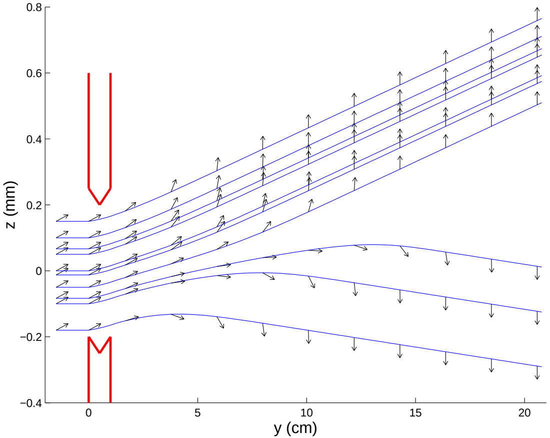
- Statistically identical to Copenhagen interpretation
- Explanation of individual impacts
- Explanation of spin quantization along the magnetic field gradient

Impacts and quantization explained by trajectories

- Initial polarization : $\theta_0 = \frac{\pi}{3}$

- Initial position : z_0

- Arrows = spin orientation : $\theta(z, t)$



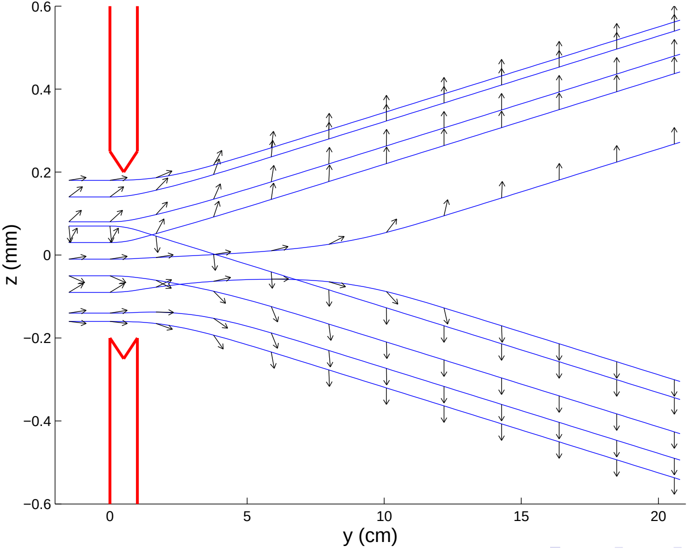
- $+\frac{\hbar}{2}$ si $z_0 > z^{\theta_0}$
- $-\frac{\hbar}{2}$ si $z_0 < z^{\theta_0}$

where $z^{\theta_0} = \sigma_0 F^{-1}(\sin^2 \frac{\theta_0}{2})$

where F is the distribution function of the standard normal distribution.

Impacts and quantization explained by trajectories

Mixed states



Impacts and quantization explained by trajectories

- **The measured position**
- **The spatial wave function**

$\Rightarrow$ postulates of quantum measurement

- adding the position of the particle to the wave function

$\Rightarrow$
- impacts
- quantization process of spin

Interaction with the measuring apparatus

- Minimum time required to measure
- Different meaning than usual

Spin orientation

- By measuring apparatus
- Depending on the position of the particle in the wave packet
- Measuring time : time required for the particle to orient its spin in its final direction.

Conclusion on quantum computer

Première remarque

Les démonstrations expliquant le gain des algorithmes de Deutsch, Glover et Shor reposent sur une factorisation des calculs utilisant des qubits intriqués [1].

factorisations approchées

Ces factorisations sont exactes pour des spineurs sans extension spatiale, mais seulement approchées pour des spineurs réels avec extension spatiale.

manque de fiabilité

C'est certainement une raison du manque de fiabilité des résultats rencontré dans les expérimentations : il y a en effet toujours une erreur quantique, attribuée habituellement à l'environnement, et qu'on essaye de corriger par des codes détecteurs d'erreurs [2].

Remarque plus fondamentale
On doit se poser l'existence même du qubit bati sur le spin d'une particule.

Le qubit indiviuel n'existe pas, seul le qubit statistique existe.

La fonction d'onde n'est pas suffisante pour représenter l'état quantique. Il faut ajouter la position de la particule. Pour représenter un qubit, il faut donc deux particules , comme en mécanique classique. cf. Chuang .

informatique quantique

Critique de l'ordinateur quantique parallèle, non de l'informatique quantique : les pbs de secret semblent bien résolus grâce à l'intrication quantique.

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